DISPERSION IN HYDROLOGIC AND COASTAL ENVIRONMENTS

by
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Final Report

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ABSTRACT

This report summarizes the results of a five-year laboratory research project on various flow phenomena of importance to transport and dispersion of pollutants in hydrologic and coastal environments. The results are useful in two general ways: first, to facilitate the prediction of ambient water quality from effluent characteristics in various water environments; and secondly, to provide the basis for design of systems (like outfalls) required to meet given ambient water quality requirements.

The results for buoyant jets may be used for the design of waste-water outfalls in oceans, reservoirs, lakes, and large estuaries. Particular emphasis is given to line sources (or slot jets) which represent long multiple-outlet diffusers, which are necessary for all large discharges to get high dilutions.

For reservoirs which are density stratified, the results include formulations for prediction of selective withdrawal, and a simulation procedure for predicting reservoir mixing by systems which pump water from one level to the other.

For application to rivers and estuaries, laboratory flume experiments were made to measure transverse mixing of buoyant or heavy tracer flows, as well as for neutral-density flows.

Abstracts for all publications and reports resulting from the project are given as an appendix to the report.

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SECTION I

CONCLUSIONS

The relationship of ambient water quality to effluent characteristics depends on a complex interaction of physical, chemical, and biological factors. This project has dealt with some fluid mechanics aspects of dispersion and transport phenomena in water environments. This report summarizes the results which are of the greatest practical value for water quality analysis and design of outfalls and other hydraulic structures for water quality control. The principal conclusions are as follows:

1. A wide range of problems in buoyant jet mechanics in stratified environments without ambient currents has been analyzed by the integral analysis originally proposed by Morton, Taylor, and Turner (1). Both two- and three-dimensional cases were solved. (See Sections IV, V, and VI.)

2. The behavior of buoyant jets in a current can be described if the ambient flow is of uniform density. Round buoyant jets in a cross flow were modelled physically and mathematically by Fan (1967, A-1 in Appendix). Slot buoyant jets in a flume (two-dimensional) present a more difficult problem because the whole regime of the flow field can be strongly affected by upstream and downstream flow conditions (i.e. whether the flow is well-mixed or two-layer, etc.). (See Section V.) Thus jet and plume formulas are probably inaccurate for application to large thermal discharges from line diffusers in restricted depths.

3. Graphs for the dilution in buoyant jets (both round and slot) in uniform environment are given in Section V as functions of a depth ratio and the Froude number. Slot jet solutions can be applied approximately for long multiport diffusers. Limiting cases of plume-like behavior can often be used for sewage discharge in deep water from a long diffuser.

4. Results for buoyant jets in stratified environment are given in abbreviated form in Section VI and in more detail in Fan and Brooks (1969, A-2). Plume solutions are found to provide a conservative approximation and are convenient for approximate solutions in cases of non-uniform density gradients.

5. In the application of all buoyant jet and plume formulas it is necessary to take account of the thickness of the pollutant field which is generated at the top of the plume. The clear height of plume rise is thus reduced, causing a lower dilution within the plume. (See Section VI.)

6. For density-stratified reservoirs an approximate analysis has been given for selective withdrawal (two-dimensional case) and for destratification by pumping. In general for a wide range of system parameters the time required for nearly complete destratification is approximately 0.2 times the reservoir volume divided by the pumping rate.

7. The dimensionless coefficient of transverse mixing of a neutrally buoyant tracer fluid in a flume flow was found to be a function of the depth-to-width ratio λ (see Fig. 28). The range was:

$$\frac{\bar{D}_z}{u_* d} = 0.093 \text{ to } 0.24$$

$$\text{for} \quad \lambda = 0.20 \text{ to } 0.093$$

where $\bar{D}_z$ is the depth-averaged transverse mixing coefficient; u_* = bed shear velocity; and d = depth. Values measured by others in the field tend to be about twice as much. The mixing coefficient includes both turbulent diffusion and lateral dispersion by secondary currents. (See Section IX.)

8. In case of a tracer stream which was buoyant or heavy with respect to the flume flow, the transverse mixing was accelerated by the density-induced secondary currents. The effect was to produce a discrete increase ($\Delta\sigma^2$) in the transverse variance of the tracer cloud at the early stages of mixing, finally adjusting to the normal rate of lateral spreading for a neutrally-buoyant tracer flow. (See Sec. IX.)

SECTION II

RECOMMENDATIONS

Additional research is recommended on the following problems within the scope of this report:

1. For the design and analysis of outfalls for effluent disposal, a method is needed for predicting the character of the transition region between the jet-mixing phase and the far-field current drift, in both uniform and stratified cases.

2. The two-dimensional jet and plume formulas and analyses are now being widely used for design of sewer outfall diffusers, but they are poorly supported by laboratory experiments compared to the three-dimensional case. In particular the entrainment coefficient (α) and the spreading ratio (λ) are subject to considerable uncertainty ($\pm 20\%$). Comprehensive laboratory experiments are needed.

3. Laboratory research is needed to determine how the dilution depends on the orientation of a line diffuser with respect to the current. In outfall design it is necessary to evaluate performance for different current directions and different possible diffuser layouts (with respect to shore).

4. Detailed field observations of the hydrodynamic performance of existing outfall diffusers are lacking and should be undertaken to confirm laboratory results and to develop any needed "scale" correction factors.

5. Experiments and analysis are needed for the case of buoyant jets and plumes in a stratified ambient current. The combination of currents and stratification is very common in the natural water bodies, but very difficult to reproduce,along with a buoyant jet, in the laboratory.

6. The fluid mechanics of mixing of thermal discharges from coastal diffusion structures in relatively limited depths needs to be studied analytically and in the laboratory. Thermal discharges are so large that they can modify the entire current and density structure in the vicinity of the diffuser. The initial dilutions obtained by jet mixing may be limited by the resistance of flow of diluting water toward and away from the diffuser.

7. The theory for selective withdrawal is based on a scale-up from laminar laboratory flows to turbulent flows in large reservoirs. Good field observations are needed to establish better values of vertical mixing coefficients and to check the validity of the theory. Three-dimensional effects of reservoirs of irregular shape also need to be studied.

8. Further research on transverse mixing of contaminants in turbulent open channel flow is needed to document and explain the differences in results between the laboratory and the field.

SECTION III

INTRODUCTION

This project, initiated in 1967, sought to bring together into a single project several flow problems affecting water quality in hydrologic and coastal environments. The fluid mechanics problems involved in dispersion in rivers, lakes, estuaries, coastal waters, and ground waters have a number of features in common to more than one area -- such as turbulence, stratified flow, Taylor dispersion and jet-induced mixing. We have tried to examine certain fundamental problems with this broad point of view.

The contributions of the research have been on a variety of topics. Since it is not feasible to report all the details on all subjects in this single final report, this document will serve as a summary of the most important practical results. The complete list of publications and reports (32 in all) is given in Section XII in chronological order. In addition, the Appendix presents abstracts for each item, arranged by topics. They are cited in this report by author, date, and abstract number in the Appendix. All items may be obtained either from the open literature, or by ordering from the W. M. Keck Laboratory of Hydraulics and Water Resources, 138-78, California Institute of Technology, Pasadena, California, 91109.

The most significant results are presented in Sections IV-IX, as follows:

Jet and plume mixing (Sections IV-VII). The mathematical models, and experiments on which they are based, provide the basis for the design of outfalls for sewage disposal, whether of single or multiple-jet type. The formulas give predictions of initial dilution, size of plumes, and maximum height of rise in case of stratified environments. Dumping of a heavy slug in a two-layer body of water is also discussed. Examples are included.

Reservoirs and lakes (Section VIII). Selective withdrawal from density-stratified reservoirs has become a widespread practice in the 1960's, at dams which are equipped with outlets at various levels (or adjustable). It is now possible to make reasonable predictions of the thickness of the withdrawal layer from the observed density stratification and the efflux rate, in order to predict the effectiveness of selective withdrawal operations at dams.

A combination of the techniques of selective withdrawal and jet mixing led to a mathematical model of a reservoir destratification technique, based on pumping water from the epilimnion and discharging it in a buoyant jet in the hypolimnion, where further mixing takes place.

Transverse mixing in rivers and other shear flows (Section IX). When contaminants are introduced into rivers,as from outfalls, there is spreading both vertically and transversely within the flow cross section. For these analyses and experiments, the mixing is mainly induced by the natural turbulence in the river itself, rather than by jet action of the discharge. The results allow for predicting the accelerated transverse mixing caused by a discharge which is either heavy or buoyant with respect to the surrounding flow.

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The text in each section is not intended as a state-of-the-art report, but rather a summary of the most usable results of this research project. A full discussion of the pertinent literature appears in each of the reports and publications. Citations to the literature (excluding items supported by this grant) are given by numerals in parentheses and are listed in Section XI. For jet and plume mixing some additional references are listed for work appearing after the project work was done.

SECTION IV

JET AND PLUME MIXING: PROBLEM DEFINITION AND METHODS OF ANALYSIS

Definitions

One of the first questions asked when an outfall sewer is being designed is how to predict the dilution achieved initially by buoyant jet mixing near the point of discharge. The initial dilution is defined as the dilution of effluent with ambient fluid at the end of this mixing process, driven by the buoyancy and momentum of the discharge. The initial dilution may be attained at the surface, or at the point of maximum height of rise in a stratified environment. If the concentration in the jet at the source is called one unit, the local dilution is just the reciprocal of concentration at any point in the flow field. The average dilution within a buoyant jet may be shown to be 1.74 times the centerline value in a round jet, and $\sqrt{2}$ times the centerline value for a two-dimensional or line buoyant jet (assuming Gaussian profiles).

A buoyant jet is a turbulent free shear flow which has both momentum and buoyancy at the source. There are two limits: if the initial momentum is small then it becomes a plume, whereas if the buoyancy is negligible it is a momentum jet. A typical buoyant jet from an outfall may be jet-like near the outlet, but become basically plume-like at some distance from the source. This is because the initial momentum gets diffused across a plume of increasing size, whereas the buoyancy continually adds new momentum to the plume as it keeps rising. Other investigators, such as Morton, use the term "forced plume" instead of "buoyant jet". In this report the plume-like rising column of fluid away from the source of a buoyant jet is often called a plume for simplicity.

A dynamic model of turbulent buoyant jets was developed following the basic integral technique of Morton, Taylor, and Turner (1) (see Fan, 1967, A-1, and Fan and Brooks, 1969, A-2). The flow pattern for a buoyant jet in a stratified fluid in the laboratory is illustrated by Fig. 1.

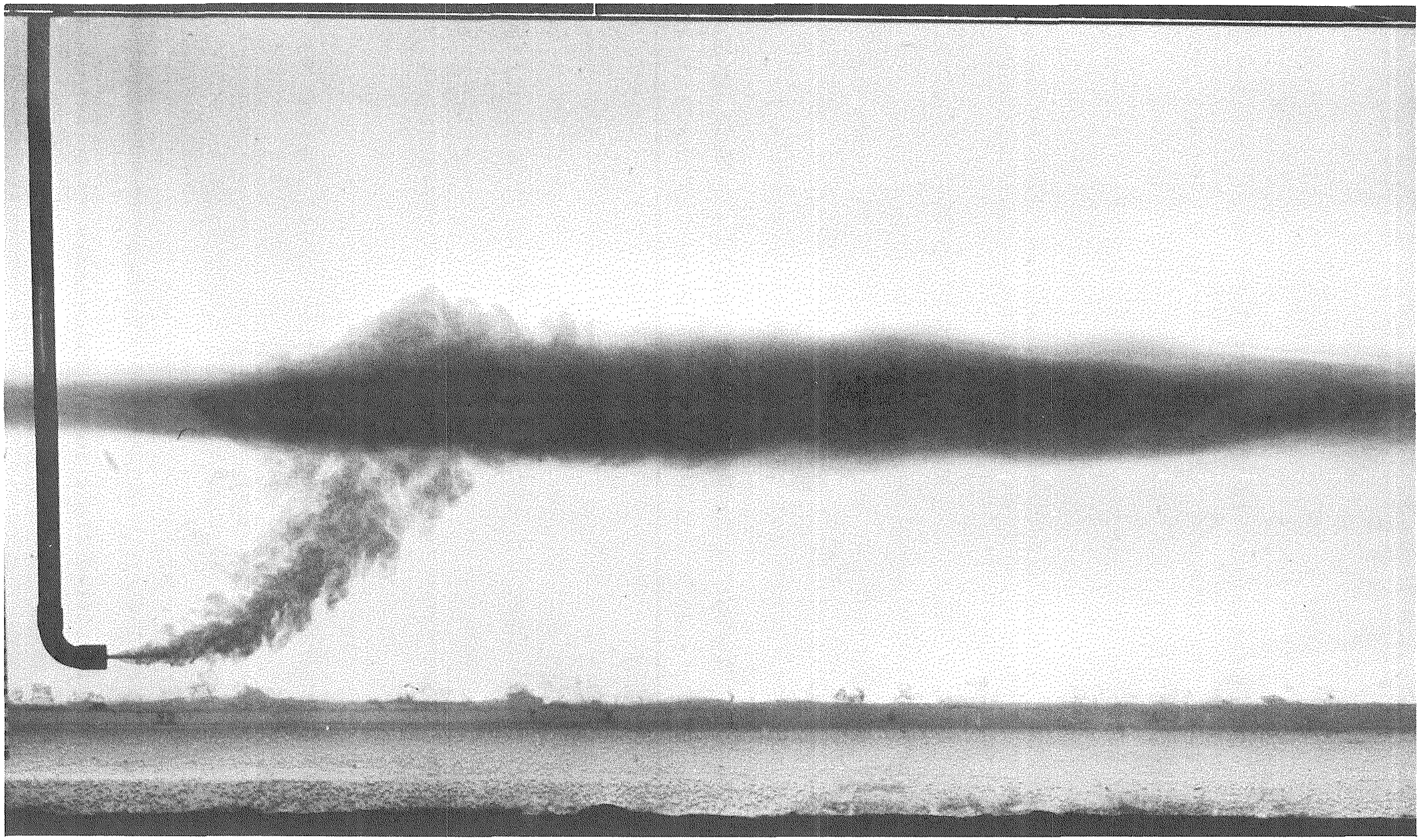


Figure 1. A buoyant jet in a laboratory tank, illustrating how the ambient density stratification prevents the jet from reaching the surface (from Fan and Brooks, 1969, A-2).

The basic definition sketches for a turbulent round buoyant jet and a slot buoyant jet are shown in Figs. 2 and 3. In each case, the upper drawing shows the case for uniform environment, wherein the plume rise is stopped only at a free surface; the lower sketches show the linearly-stratified cases where the stratification limits the height of rise of the plumes. Only the assumptions will be carefully detailed here, with the reader being referred to the original reports for the details of the solutions.

General Assumptions

The general assumptions underlying the analyses made in this investigation are listed as follows:

1. The fluids are incompressible.

2. Variations of fluid density throughout the flow field are small compared with the reference density chosen. The variaton of density can be neglected in considering inertia terms but it must be included in gravity terms. As the variations in density are assumed small, separate conservation equations can be written for volume flux and buoyancy flux (or flux of the agent causing density change, i.e., heat or salts). This is commonly called the Boussinesq assumption.

3. Within the range of variation, the density of the fluid is assumed to be a linear function of either salt concentration or heat content above the reference level.

4. The flow is fully turbulent. Molecular transport can be neglected in comparison with turbulent transport. There is no Reynolds number dependence.

5. Longitudinal turbulent transport is small compared with longitudinal advective transport.

6. Pressure is hydrostatic throughout the flow field.

7. Curvature of the trajectory of the jet is small. In other words, the ratio of the local characteristic width of the jet to the radius of curvature is small. The effect of curvature will be neglected.

8. The velocity profiles are similar at all cross sections normal to the jet trajectory. Similarity is also presumed for profiles of

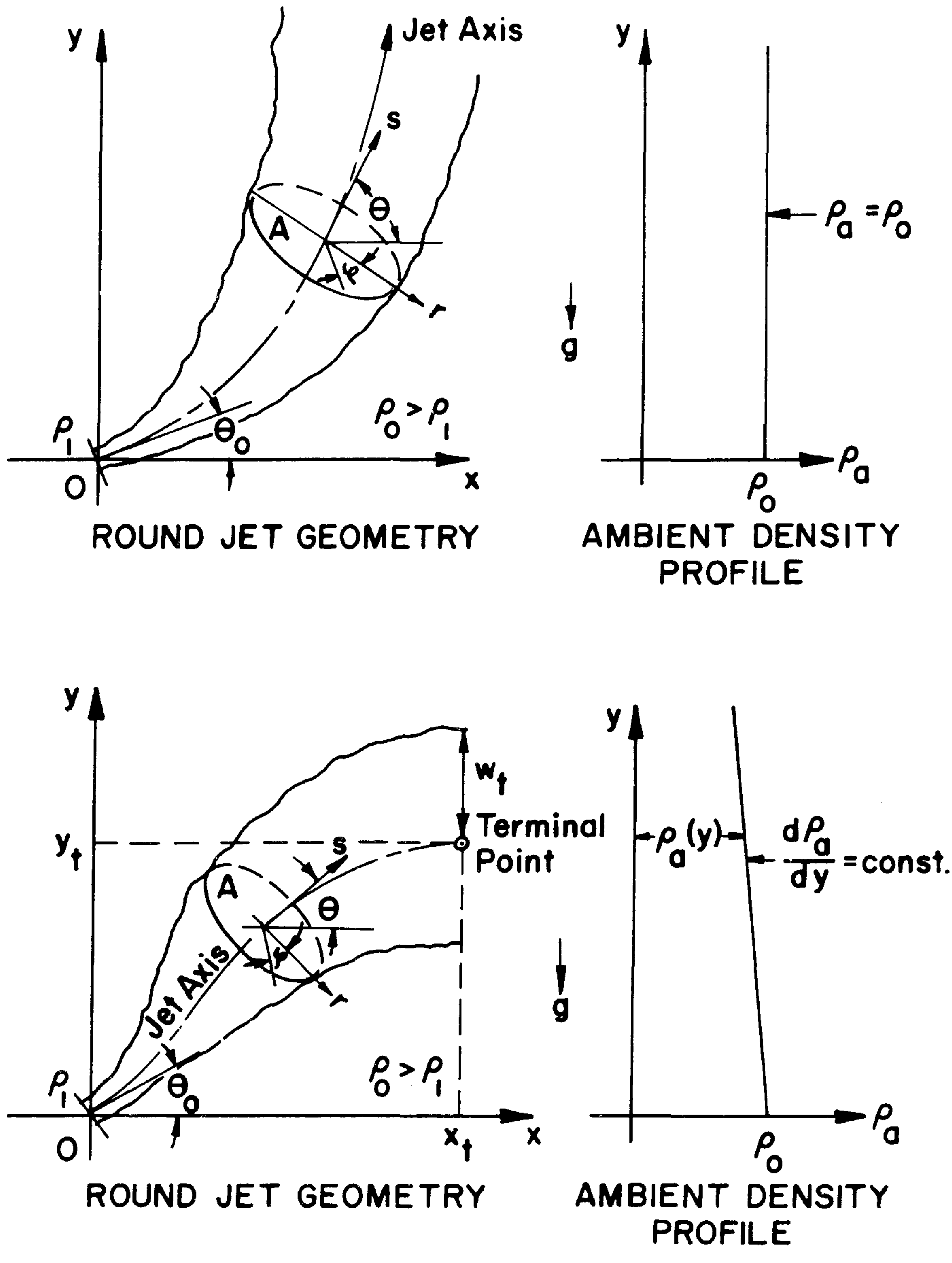


Figure 2. Schematic diagrams of round buoyant jet problems studied. Top: uniform environment; bottom: linear density stratification.

buoyancy and concentration of any tracer. The specific forms of the profiles are given below. The analyses apply only to the zone of established flow where all the profiles are fully developed. However, for practical applications, the initial conditions must be adjusted to take account of the zone of flow establishment. The results by Albertson, Dai, Jensen and Rouse (2) for this region will be adopted in application to practical problems.

Assumptions for An Inclined Round Buoyant Jet in a Stagnant Environment -- Uniform or with Linear Density-Stratification

In Fig. 2 the jets are issuing from the origin at an angle of inclination θ_o with the horizontal. The axis of the jet is taken as a parametric coordinate axis s. The angle between the s-axis and the horizontal is denoted as θ. The radial distance to the s-axis at a normal cross section A is chosen to be the r-coordinate. The angular coordinate φ is denoted as shown in the figures.

u^* and ρ^* are respectively local mean velocity and density, which are functions of r and s, while u and ρ are the characteristic velocity and density at the s-axis and are functions only of s. Axial symmetry is presumed, allowing no dependence on φ. The corresponding ambient density values are similarly denoted as $\rho_a{}^*$ and ρ_a. In a uniform ambient fluid $\rho_a{}^* = \rho_a$ = constant.

In a uniform environment, the jet axis is deflected upwards because of the increase of vertical momentum flux due to the action of the buoyancy force. The jet grows as it rises and entrains ambient fluid.

In a linearly density-stratified environment ($d\rho_a/dy$ = constant), the jet axis is first deflected upwards because of the increase of vertical momentum flux due to the action of the buoyancy force. Because of the turbulent mixing, the jet entrains the denser ambient fluid and grows heavier with reduction of the driving buoyancy force. Since the density of the ambient fluid decreases with height, the jet will eventually become as heavy as, and then heavier than, the ambient fluid at the same height. The buoyancy force thus reverses its direction and in the end will stop the rising of the jet at a terminal point (x_t, y_t) where the vertical momentum flux vanishes. The trajectory of the jet is therefore in general an S-shaped curve. After reaching the terminal point the horizontal momentum flux (if any) will keep the jet moving in the x-direction. But the flow cannot maintain the characteristics of a turbulent jet after reaching the terminal level and collapses in the vertical direction because of the suppression of vertical motion imposed by the density stratification. The analysis does not cover that part.

Specific assumptions related to the analyses of round buoyant jet problems are listed as follows:

1. The entrainment relation is given by the equation:

$$\frac{dQ}{ds} = 2\pi\alpha b u \qquad (1)$$

where Q is the volume flux across the jet cross section A; α is a coefficient of entrainment for a round buoyant jet; b is the characteristic length defined in Eq. 2; u is the characteristic velocity along s-axis.

2. Velocity profiles are assumed to be Gaussian:

$$u^*(s,r) = u(s)\; e^{-r^2/b^2} \qquad (2)$$

where b = b(s) is a characteristic length defined by the velocity profile. Commonly $w = \sqrt{2}\, b$ is defined to be the nominal half-width of the jet.

3. Profiles of density deficiency with respect to the ambient density are assumed to be Gaussian:

$$\frac{\rho_o - \rho^*(s,r)}{\rho_o} = \frac{\rho_o - \rho(s)}{\rho_o}\; e^{-r^2/(\lambda b)^2}$$

(in a uniform environment) (3a)

$$\frac{\rho_a^*(s,r) - \rho^*(s,r)}{\rho_o} = \frac{\rho_a(s) - \rho(s)}{\rho_o}\; e^{-r^2/(\lambda b)^2}$$

(in a linearly density-stratified environment) (3b) where λb is the characteristic length of the profiles; λ^2 is the turbulent Schmidt number which is assumed to be constant and is usually found to be somewhat larger than 1. Such profiles can also be regarded as buoyancy profiles.

4. Concentration profiles for passive tracers (such as trace metals, bacteria, etc.,which do not affect density) are also similar and assumed to be Gaussian:

$$c^*(s,r) = c(s)\, e^{-r^2/(\lambda b)^2} \tag{4}$$

<u>Assumptions for An Inclined Two-Dimensional Slot Buoyant Jet in a Stagnant Environment -- Uniform or with Linear Density Stratification</u>

In Fig. 3, the jets are issuing from the z-axis at an angle of inclination θ_o with the horizontal. The axis of the jet is again taken as a parametric coordinate axis s. The distance normal to the s-axis is taken to be the n-coordinate as shown.

The flow configurations are entirely similar to those described above. Specific assumptions related to the analyses of two-dimensional slot jet problems are listed as follows:

1. The entrainment relation is given by the equation:

$$\frac{dq}{ds} = 2\alpha u \tag{5}$$

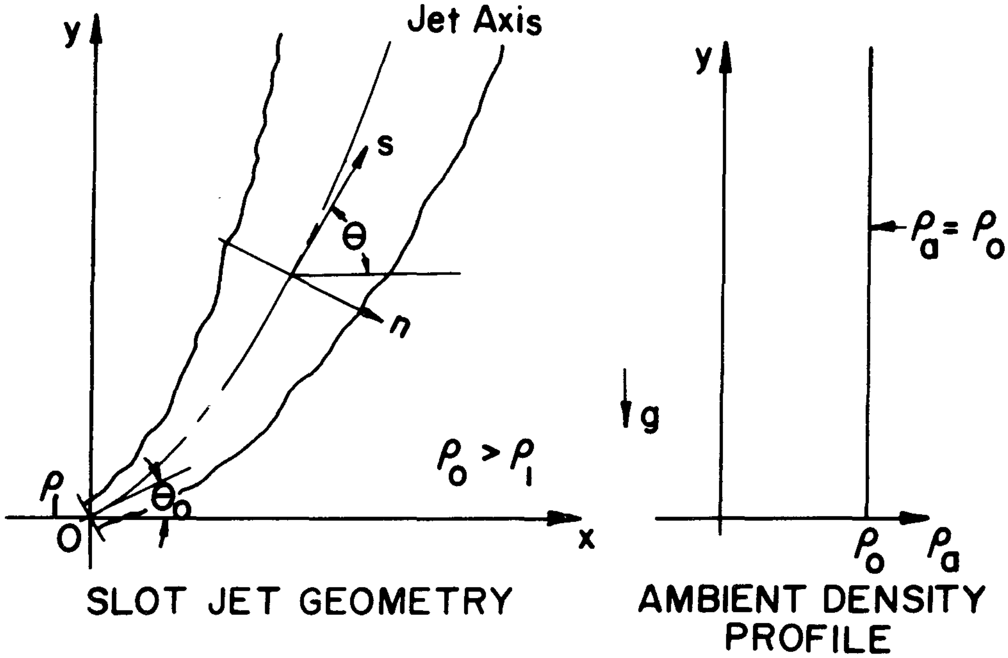
where q is the volume flux per unit length along z-axis; α is an entrainment coefficient for a slot buoyant jet.

2. Velocity profiles are assumed to be Gaussian:

$$u^*(s,n) = u(s)\, e^{-n^2/b^2} . \tag{6}$$

$\sqrt{2}\, b$ is again defined to be the nominal half width of the jet.

3. Profiles of density deficiency with respect to the ambient density are assumed to be Gaussian:



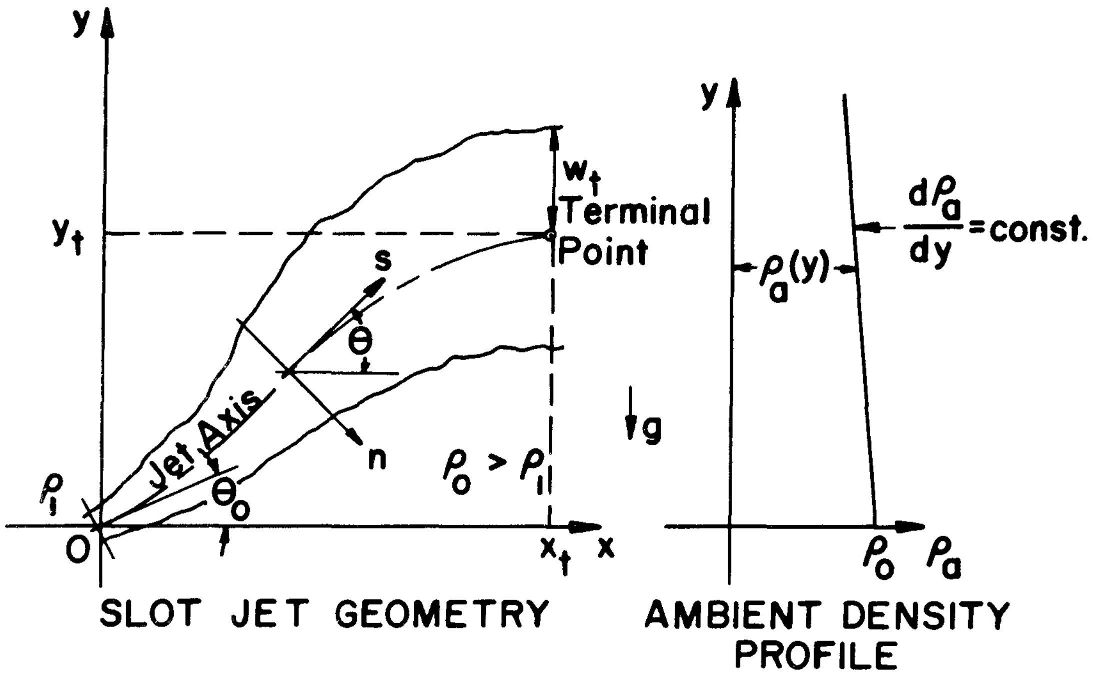


Figure 3. Schematic diagrams of slot buoyant jet problems studied. Top: uniform environment; bottom: linear density stratification.

$$\frac{\rho_o-\rho^*(s,n)}{\rho_o} = \frac{\rho_o-\rho(s)}{\rho_o} e^{-n^2/(\lambda b)^2}$$

(in a uniform environment) (7a)

$$\frac{\rho_a^*(s,n) - \rho^*(s,n)}{\rho_o} = \frac{\rho_a(s)-\rho(s)}{\rho_o} e^{-n^2/(\lambda b)^2}$$

(in a linearly density-stratified environment) (7b)

4. Profiles of passive-tracer concentration are also Gaussian:

$$c^*(s,n) = c(s)\, e^{-n^2/(\lambda b)^2} \tag{8}$$

Method of Solution

Using the above assumptions the following basic differential equations are written:

1. Entrainment (conservation of volume)
2. Horizontal momentum flux (conserved)
3. Vertical momentum flux (changes due to jet buoyancy)
4. Buoyancy flux (conserved in uniform environment only)
5. Horizontal displacement
6. Vertical displacement
7. Passive tracer conservation

The equations are integrated across the plumes so that the system reduces to only one independent variable, s, the distance along the axis. The initial conditions include initial volume flux, momentum flux and buoyancy flux. The environment may be uniform or linearly-stratified. A large number of normalized computer solutions were made, as reported in Fan and Brooks (1969, A-2). On the basis of these solutions, contour graphs were prepared to summarize the results; some of the most useful graphs are given in the next sections.

All of the mathematical modelling applies only to the zone of established flow, that is the zone beyond the distance of about 6D for round jets (see Fig. 4). Within this short distance from the source, the boundary layers at the edges of the jet are still growing; the similarity assumptions used in the analysis do not apply until these shear layers reach the center of the jet. In this initial zone of flow establishment the centerline velocity stays at u_o, the initial value and only starts declining by jet diffusion beyond a distance of 6D.

However, the centerline concentration at the end of the zone of flow establishment is already lower by the factor $(1+\lambda^2)/2\lambda^2$ for round jets, because scalar quantities spread slightly faster. The ratio λ is taken as 1.16 (based on experiments); therefore $(1+\lambda^2)/2\lambda^2 = 0.87$. In all of the following results it is necessary to make slight adjustments to change the basis of the solution from the end of the zone of flow establishment to the beginning of the jet at the nozzle.

The values of the entrainment coefficient (α) and the spreading ratio (λ) used are as follows (Fan and Brooks, 1969, A-2):

	α	λ
Round jets	0.082	1.16
Slot jets	0.14	1.00

In the original Fan and Brooks report, the results are given in a general way, which permits the user to select values of α and λ, although they are restricted to constants.

The values given above for the slot jet ($\alpha = 0.14$, $\lambda = 1.00$) are revised from those given by Fan and Brooks (1969, A-2) ($\alpha = 0.16$, $\lambda = 0.89$). The new values come from a redrawing of the curves to fit the experimental points better in the principal reference by Rouse, Yih, and Humphreys (3). The previous values of α and λ were based on the curves drawn by them. The reader is cautioned that in any event the coefficients are poorly known and based on scanty evidence for the two-dimensional case.

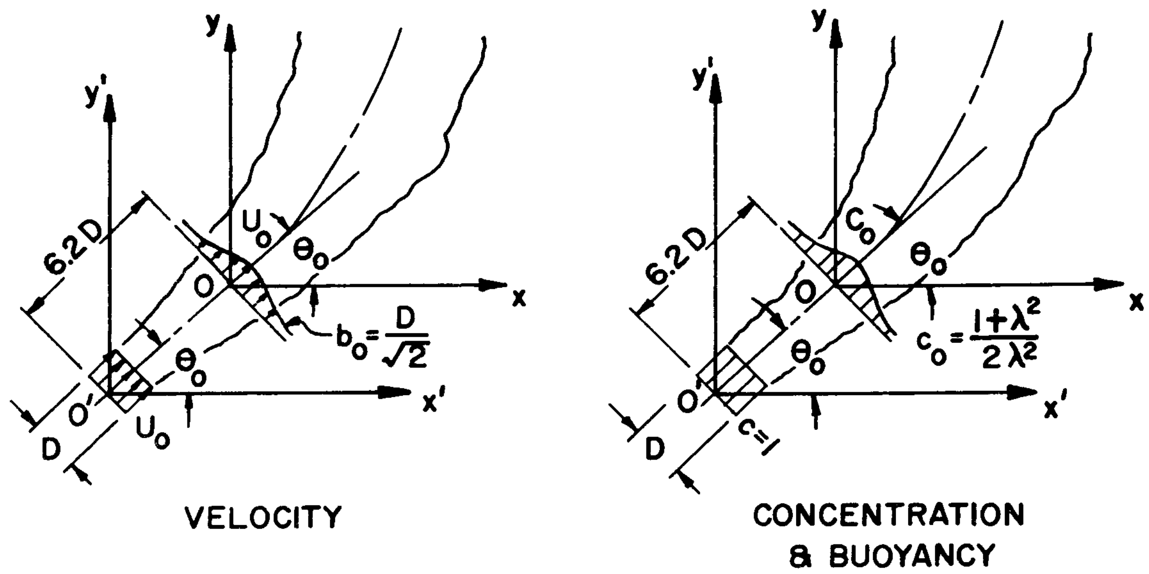


Figure 4. Zone of flow establishment for an inclined round buoyant jet.

List and Imberger (1973, A-12) have recently shown theoretically how the two coefficients must be related to each other, and how α for a buoyant jet is really a variable dependent on local Froude number, with asymptotes of 0.057 for the pure axisymmetric jet and 0.082 for the pure axisymmetric plume. The integrations made in the earlier report did not take this variation into account, but the errors are not considered large unless the flow is more jet-like than plume-like. In most practical cases of sewage discharge in the ocean, the buoyancy effects are dominant, and the flows tend to be plume-like. Further work is needed.

SECTION V

RESULTS FOR TURBULENT BUOYANT JETS IN UNIFORM ENVIRONMENTS

This section gives the most important results for turbulent buoyant jets discharging either horizontally or vertically into uniform ambient environments, without ambient currents. The full details of the solutions are available in the report by Fan and Brooks (1969, A-2).

The case of discharge of a round buoyant jet into an ambient current is not covered here but it is treated both analytically and experimentally by Fan (1967, A-1). Some results by Cederwall (1971, A-6) for flow regimes for slot jets into a current are included.

Further work is in progress for developing other simplified results for use in design problems.

Round Buoyant Jet (Uniform Environment With No Current)

The analytical solutions for the dilution on the centerline of a round buoyant jet in a uniform environment without current can be represented (as shown in Figs. 5 and 6) by a function

$$S_o = f(y/d,\ F,\ \theta) \tag{9}$$

S_o = centerline (or minimum) dilution in the buoyant jet (relative to concentration on centerline at end of zone of flow establishment)

where y = vertical distance from center of jet at end of flow establishment to the point of measurement of S_o in center of plume

D = diameter of jet at source (at vena contracta if there is jet contraction)

$$F = \frac{Q}{\frac{\pi}{4}D^2\sqrt{\frac{\rho_o-\rho_1}{\rho_o}\,gD}} = \frac{U_o}{\sqrt{\frac{\rho_o-\rho_1}{\rho_o}\,gD}} \tag{10}$$

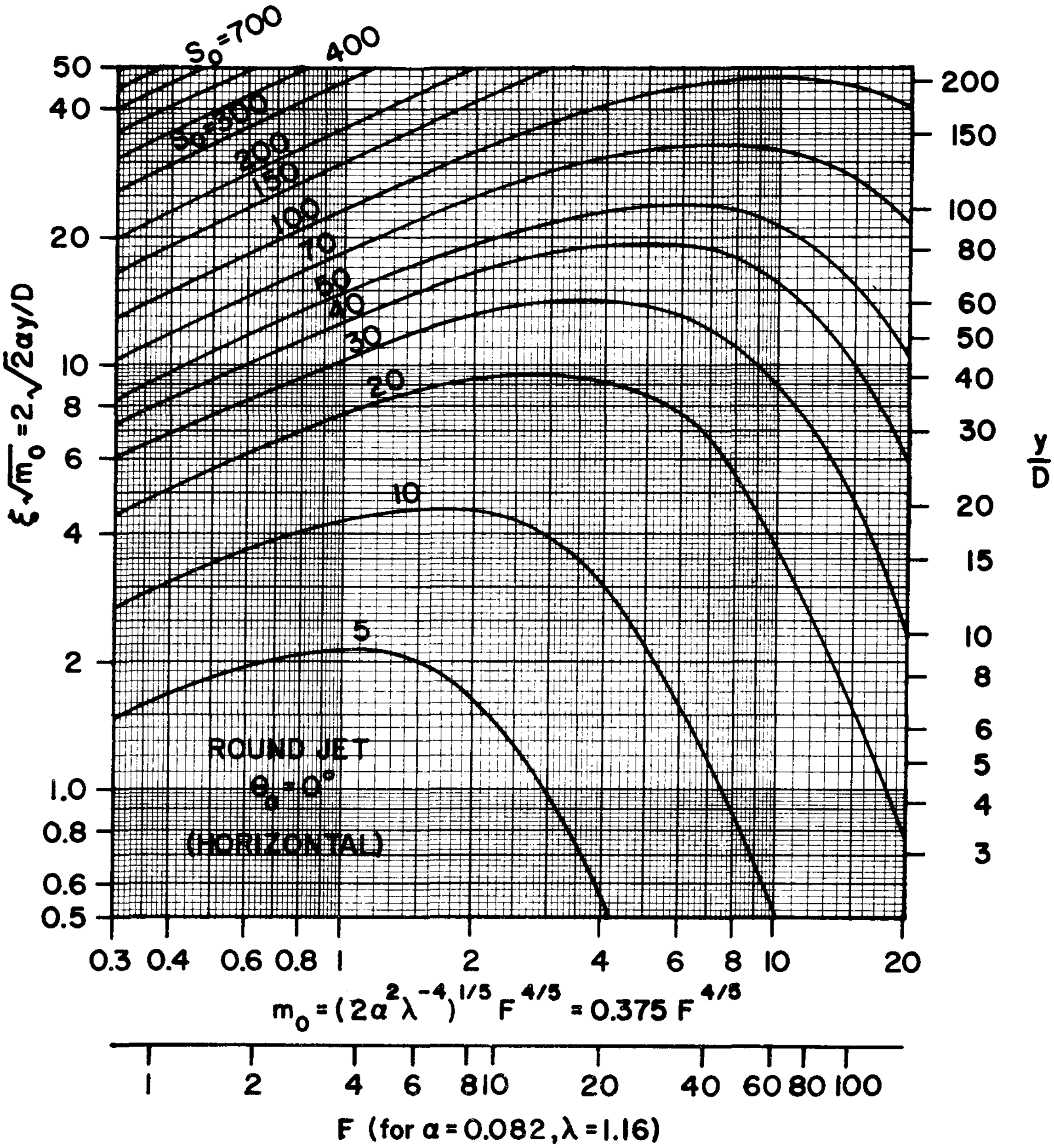


Figure 5. Centerline dilution of round buoyant jets in stagnant uniform environments: $\theta_o = 0^o$ (horizontal). To get centerline dilution relative to the nozzle, multiply S_o by 1.15 to adjust for the zone of flow establishment; for average multiply by 2. (After Fan and Brooks, 1969, A-2.)

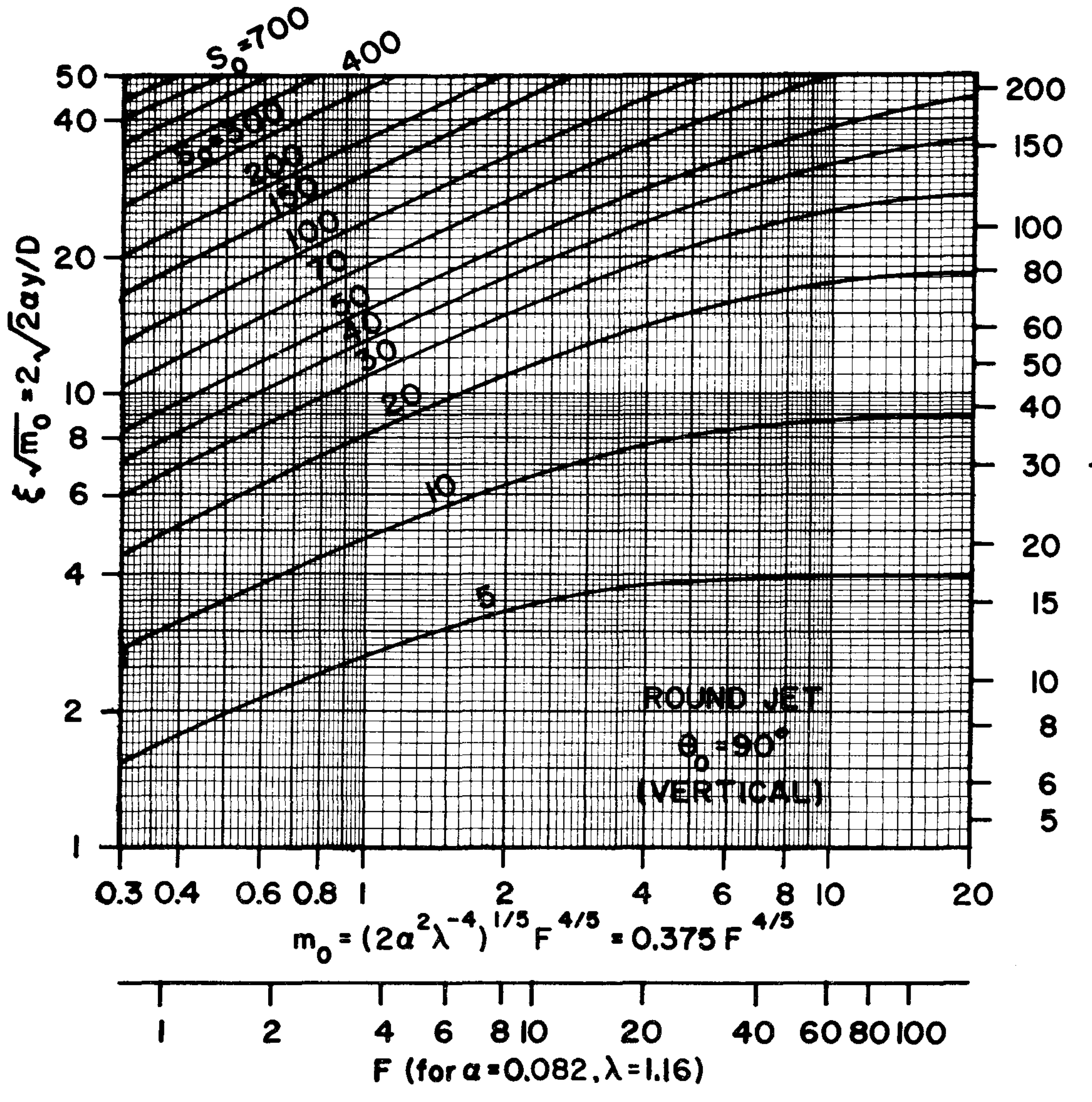


Figure 6. Dilution of round buoyant jets in stagnant uniform environments: $\theta_o = 90^o$ (vertical). To get dilution relative to the nozzle, multiply S_o by 1.15 to adjust for zone of flow establishment; for average multiply by 2. (After Fan and Brooks, 1969, A-2.)

Q = discharge in initial jet

U_o = initial jet velocity

g = acceleration of gravity

ρ_o = reference density = density of ambient fluid

ρ_1 = density on the centerline at the end of the zone of flow establishment

θ = angle of discharge ($\alpha = 0^o$ for horizontal; $\theta = 90^o$ for vertical).

Other parameters shown on the graphs ($\zeta\sqrt{m_o}$ and m_o) relate to the theoretical solutions and are not needed for application of the results in terms of the above parameters.

The analytical results given here are based on α = entrainment coefficient = 0.082, and λ = spreading ratio = 1.16. The dilutions S_o are referred to the end of the zone of flow establishment, and must be increased by the factor $2\lambda^2/1+\lambda^2 = 1.15$ (for $\lambda = 1.16$) for practical application for referring the dilutions to the beginning of the jet (see Fig. 4). In other words, if S_{od} is the dilution referred to the concentration of the initial discharge

$$S_{od} = \frac{2\lambda^2}{1+\lambda^2} S_o = 1.15\ S_o \quad \text{for } \lambda = 1.16 \tag{11}$$

To obtain the average dilution S_a, it is first necessary to find the average concentration $\bar{c}$ of a tracer across a plume weighted with flow velocity to give a volumetric average:

$$\bar{c}(s) = \frac{\int_o^\infty u(s)\ e^{-r^2/b^2}\ c(s)\ e^{-r^2/(\lambda b)^2} 2\pi r dr}{\int_o^\infty u(s) e^{-r^2/b^2}\ 2\pi r dr} = \frac{\pi b^2\ \lambda^2/(1+\lambda^2)}{\pi b^2} c(s)$$

$$= \frac{\lambda^2}{1+\lambda^2} c(s)$$

$$S_a = \frac{1+\lambda^2}{\lambda^2} S_o\ . \tag{12}$$

For S_{ad}, the average dilution referred to the original jet:

$$S_{ad} = \frac{2\lambda^2}{1+\lambda^2} S_a = 2S_o \ . \tag{13}$$

Thus to obtain the average dilution of the plume relative to the initial jet, multiply the graph values of S_o by 2 for round jets.

The same type of adjustment for the zone of flow establishment applies for the density difference $[\rho_o - \rho(s)]$ as for tracer concentration $c(s)$, because the spreading rate for the two is assumed to be the same by Eqs. 3a and 4.

Note that the Froude number F used in this work is based on the relative density difference, $(\rho_o-\rho_1)/\rho_o$, at the end of the zone of flow establishment. If the jet has an initial density of ρ_d at the point of discharge, then the conservation equations for the zone of flow establishment yield (see Fig. 4):

$$\frac{\rho_o-\rho_1}{\rho_o} = \frac{\rho_o-\rho_d}{\rho_o} \cdot \frac{1+\lambda^2}{2\lambda^2} = 0.87 \ \frac{\rho_o-\rho_d}{\rho_o} \quad \text{for } \lambda = 1.16 \ . \tag{14}$$

Alternatively, if the value of the source Froude number F_d is known, defined as

$$F_d = \frac{U_o}{\sqrt{g\left(\frac{\rho_o-\rho_d}{\rho_o}\right)D}} \ , \tag{15}$$

the conversion equation is

$$F = 1.07 \ F_d \ . \tag{16}$$

This difference is scarcely significant, nonetheless the distinction is made for consistency. This distinction was not made in the Fan and Brooks report, as they incorrectly referred to ρ_1 as the initial jet density rather than the density at the end of the zone of flow establishment.

The distance from the beginning of the jet to the end of the zone of flow establishment is approximately 6D. For vertical discharges the real dimensionless distance y/D must be reduced by 6 before entering the graphs (Figs. 6 and 8).

In the theory the distance y is the vertical coordinate in the solutions for buoyant jets which are presumed to rise indefinitely. In practice we are often interested in the dilution at the point where the rising plume reaches the water surface. Although the flow pattern is deflected by the surface, the dilution calculated is often based on setting y = total depth; however, a more conservative approach would be to use the effective depth y somewhat less than the total depth (perhaps reduced by an amount equal to one quarter of the plume diameter at the top).

Toward the upper left of both graphs (Figs. 5 and 6), the solutions become identical, as the results for both cases are asymptotic to the solution for a simple buoyant plume. For $\frac{y/D}{F} > 30$, the plume solution for centerline dilution may be used as follows:

$$S_o = 0.095 \ (y/D)^{5/3} \ (F)^{-2/3} \ . \tag{17}$$

Referred to the beginning of the jet (multiplying by 1.15):

$$S_{od} = 0.109 \ (y/D)^{5/3} \ F^{-2/3} = 0.089 \ \frac{g'^{1/3} y^{5/3}}{Q^{2/3}} \ , \tag{18}$$

where $g' = g \frac{\rho_o - \rho_d}{\rho_o}$. Since $F \sim D^{-5/2}$ for given Q, S_o is actually independent of D (and the initial velocity) for plume-like behavior. For many applications (including extrapolation to values of y/D off the top of the graphs), the plume solution, Eq. 18, is entirely adequate.

The half-width of the jets is defined as two standard deviations of the transverse velocity distribution, or $w = \sqrt{2}\ b$, where b is defined in Eq. 2 as

$$u^*(s,r) = u(s) \ e^{-r^2/b^2} \ . \tag{19}$$

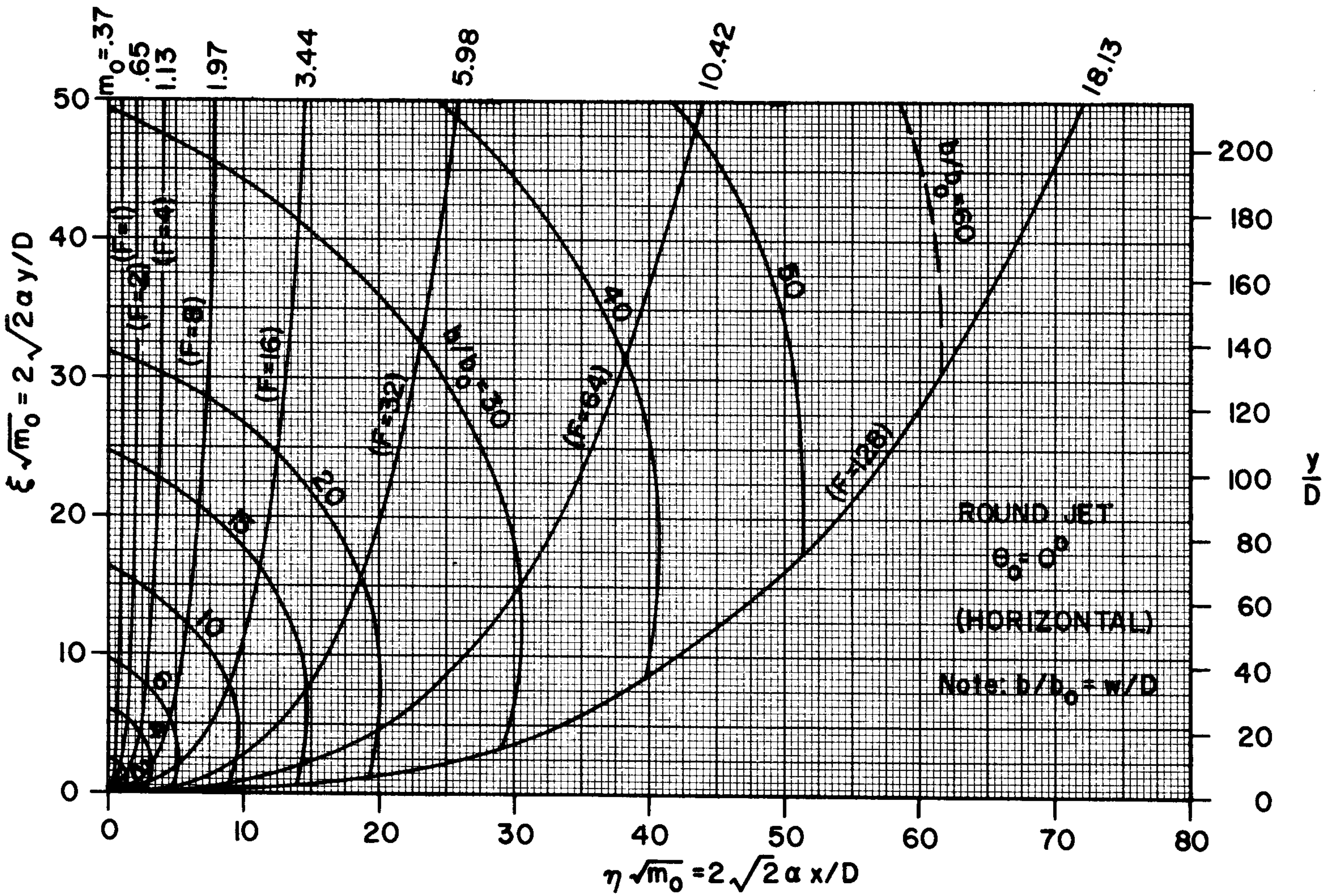


Figure 7. Trajectory and half-width b/b_o of round buoyant jets in stagnant uniform ambient fluids: $\theta_o = 0^o$ (horizontal). (F and y/D scales based on $\alpha = 0.082$ and $\lambda = 1.16$.) (After Fan and Brooks, 1969, A-2.)

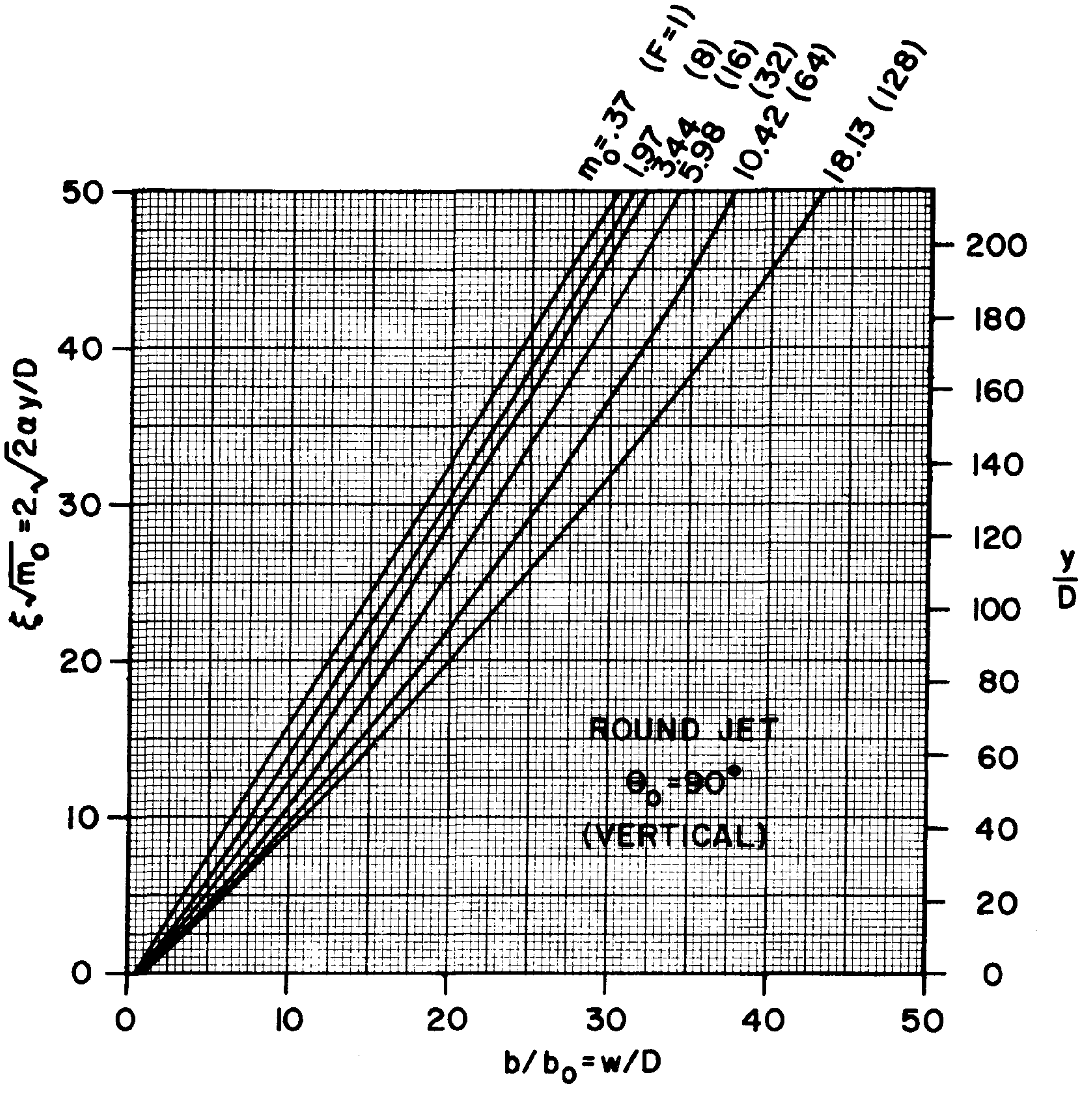


Figure 8. Half-width b/b_o of round buoyant jets in stagnant uniform ambient fluids: $\theta_o = 90°$ (vertical). Trajectories are all vertical lines. (F and y/D scales based on α = .082 and λ = 1.16.) (After Fan and Brooks, 1969, A-2.)

At the end of the zone of flow establishment, $b = b_o$; by conservation of momentum, b_o may be related to the initial jet diameter D by the relation $b_o = D/\sqrt{2}$.

Fig. 7 gives dimensionless trajectories for various Froude numbers, overlaid with contours of $w/D = b/b_o$. For vertical buoyant jets, the trajectories are all the same; therefore Fig. 8 gives only the widths w/D for various F values. To obtain the full width (2w), multiply values of w/D read from either graph by 2D.

The simple-plume result for width is

$$b = 0.102y \tag{20}$$

or for total width, 2w

$$2w = 2\sqrt{2}\ b = 0.29y\ . \tag{21}$$

The reader is cautioned that the results given here are all based on the entrainment coefficient $\alpha = 0.082$. This value is appropriate for flows which are plume-like and does not allow for smaller α-values near the source where the flows may be more jet-like. List and Imberger (1972, A-12) studied this problem at the very end of the grant period and developed a rationale for the transition of α-values from jet to plume regions. Furthermore, even for pure plumes the α-value is only known within $\pm$ 15%. Therefore, the results obtained should not be considered more accurate than $\pm$ 15 to 20%.

Example. Consider a discharge of Q = 50 mgd of sewage effluent from a diffuser at a depth of 65 feet in homogeneous sea water (no stratification). There is sufficient head to jet the sewage out at 15 fps. The density of sea water is 1.025 gr/ml and sewage is 0.999 gr/ml. Compare initial dilutions which can be obtained by a 50-port diffuser with a 5-port one. Assume ports are rounded inside and produce no jet contraction, and are separated sufficiently to avoid interference. The jet discharge is horizontal.

Required total area of jets $= \dfrac{50 \times 1.55 \text{ cfs}}{15 \text{ ft/sec}} = 5.17 \text{ ft}^2$

a. For 50 ports:

$$\frac{\pi}{4} D^2 = \frac{5.17}{50} = 0.1034 \text{ ft}^2$$

$$D = 0.362 \text{ ft} = 4.35 \text{ in.}$$

$$y/D = 65/0.362 = 180$$

$$F_d = \frac{U_o}{\sqrt{\frac{\Delta\rho}{\rho} gD}} = \frac{15}{\sqrt{(.026)\ (32.2)\ (.362)}} = 27$$

$$F = 1.07\ F_d = 29$$

By Fig. 5, $S_o = 100$

$S_{od} = 115$ = dilution on centerline at top of plume (by Eq. 11)

$S_{ad} = 200$ = average dilution (by Eq. 13)

b. For 5 ports:

$$\frac{\pi}{4} D^2 = \frac{5.17}{5} = 1.034 \text{ ft}^2$$

$$D = 1.15 \text{ ft} = 13.75 \text{ in.}$$

$$y/D = 65/1.15 = 57$$

$$F_d = \frac{15}{\sqrt{(0.026)(32.2)\ (1.15)}} = 15.2$$

$$F = 1.07\ F_d = 16.3$$

By Fig. 5, $S_o = 28$;

$S_{od} = 32$

$S_{ad} = 56$.

All solutions assume no interference between rising plumes. For case (a) (50 ports), Fig. 7 gives w/D = 35 at y/D = 180 for F = 29; therefore, the diameter at the head of the plume is approximately 2w = 2 x 35 (0.36 ft) = 25 ft. Similarly, for case (b) (5 ports), we find w/D = 13 for y/D = 57 and F = 16; therefore, the diameter at the top of the plume is predicted to be about 2w = 2 x 13 x 1.15 ft = 30 ft.

Slot Buoyant Jet (Uniform Environment With No Current)

The analytical solutions for a slot buoyant jet are useful for multiport line diffusers in which a row of ports giving jet diameter D at spacing s may be considered equivalent to a slot giving a jet of width B. If the ports are spaced closely enough so that there is extensive interference, then the row of jets will produce a flow pattern similar to that of a slot jet when viewed from a moderate distance. The equivalent width of a slot jet is based on providing the same flux of momentum, volume and buoyancy; therefore the areas must be equal or

$$\frac{\pi}{4} D^2 = Bs,$$

$$\text{or} \qquad B = \frac{\pi}{4} \frac{D^2}{s} \quad . \tag{22}$$

When designing a line diffuser the important variable is jet area per foot or B; the spacing s may be chosen small compared to the total depth in order to realize the total benefit of a line source. However, excessively close spacing or the use of an actual slot is unnecessary. When comparing different possible port spacings (s), the port size must be modified in accordance with $D \propto \sqrt{s}$ in order to keep the same basic flux rates per unit length of diffuser. If the jets are so far apart that they act as individual round buoyant jets without interference, then they are farther apart than necessary; in other words, a better solution might be to use smaller, closer jets which do interfere and thus better approximate a line source.

Fig. 9 gives the analytical solution for horizontal slot buoyant jets. As for the round jets, the dilution can be represented as a function of the Froude number and the depth ratio y/B. In this case, the Froude number is

$$F = \frac{U_o}{\sqrt{g\left(\frac{\rho_o - \rho_1}{\rho_o}\right)B}} \quad . \tag{23}$$

But since we now estimate $\lambda = 1.0$ for two-dimensional buoyant jets (rather than $\lambda = 0.89$ as given by Fan and Brooks), no correction is

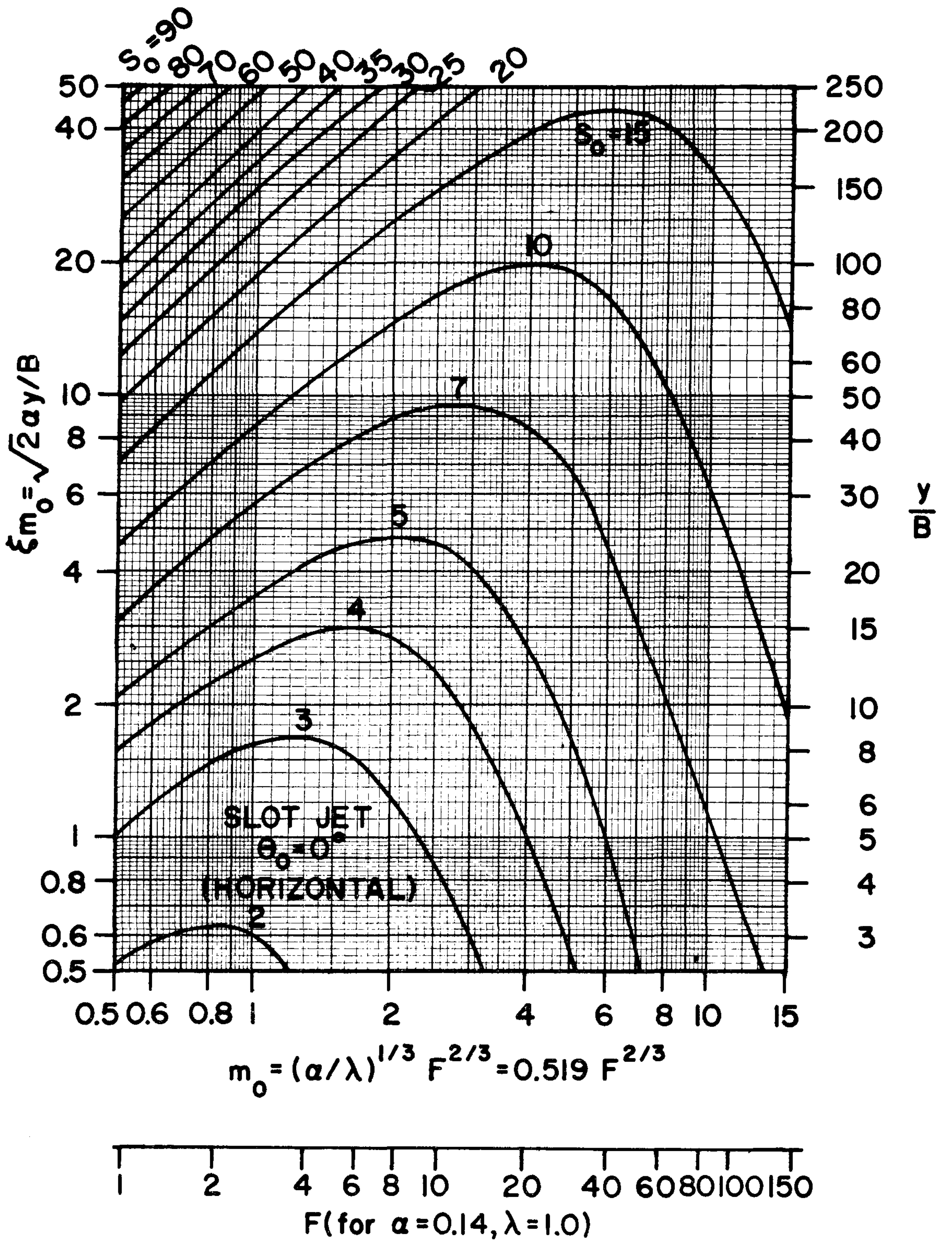


Figure 9. Centerline dilution of slot buoyant jets in stagnant uniform environments: $\theta_o = 0^o$ (horizontal). (For average dilution, multiply by $\sqrt{2}$.) (After Fan and Brooks, 1969, A-2.)

necessary for the zone of flow establishment, and $(\rho_o - \rho_1) = (\rho_o - \rho_d)$ at the nozzle. The dilutions S_o given in the graphs are correct with respect to the original jet.

The average dilution in the rising plume is again found from the weighted average of the concentration:

$$\bar{c}(s) = \frac{\int_{-\infty}^{\infty} u(s)\ e^{-n^2/b^2} c(s) e^{-n^2/b^2}\ dn}{\int_{-\infty}^{\infty} u(s) e^{-n^2/b^2}\ dn}$$

$$= \frac{c(s)}{\sqrt{2}} .$$

Therefore $$S_a = \sqrt{2}\ S_o . \tag{24}$$

The distance from the beginning of the jet to the end of the zone of flow establishment is approximately 5B. This correction is rarely necessary because B is small in most practical cases.

For the upper left portion of Fig. 9 the dilutions are asymptotic to those for a line plume. For $(y/B)F^{-4/3} > 20$ (see Cederwall, 1971, A-6, p. 28), the plume solution for centerline dilution may be used:

$$S_o = 0.38\ (y/B)F^{-2/3} \tag{25}$$

or $$S_o = 0.38\ g'^{1/3} y q^{-2/3} \tag{26}$$

where q = discharge per unit length

and $g' = g \frac{\rho_o - \rho_d}{\rho_o} = g \frac{\rho_o - \rho_1}{\rho_o}$.

For the plume approximation, the dilution is independent of B as it should be; only the buoyancy flux is important and not the momentum. Since almost all practical cases fall within the plume approximation, for which the angle of discharge is not a variable, the graph for a vertical buoyant slot jet is not reproduced here (see Fan and Brooks, 1969, A-2).

In case $(y/B)F^{-4/3} < 20$, but the solution point is still off the graph, the solution may be obtained as follows. By similarity arguments, or by examination of the functional relation in Fig. 9, the dilution may be written approximately as

$$S_o F^{-2/3} = f(\frac{y}{B} F^{-4/3}) \tag{27}$$

Therefore a similar point in the graph may be found for use in evaluating the function f (see example below).

For the plume solution, the size may be taken as

$$b = 0.16y \tag{28}$$

$$2w = 2\sqrt{2}\ b = 0.45y. \tag{28a}$$

Example. Consider the previous example for round buoyant jets, case (a), 50 ports. Assume a line diffuser of 50 ports at 25-foot spacing, and compute the plume dilution and width by slot-jet analysis.

$$B = \frac{\pi}{4}\frac{D^2}{s} = \frac{0.1034\ ft^2}{25} = 0.00414\ ft$$

$$y = 65\ ft$$

$$y/B = 65/0.00414 = 15{,}700$$

$$F = \frac{15\ fps}{\sqrt{(0.026)\ (32.2)\ (.00414)}} = 255 \qquad \text{by Eq. 23.}$$

Since this point is off the graph (Fig. 9), find the value of

$$(y/B)F^{-4/3} = 15{,}700/(255)^{4/3} = 9.7.$$

This ratio is less than 20, so we must use the similarity relation, Eq. 27. Choose a similar point $(y/B)' = 200$; solve for the similar F' from the relation

$$(y/B)'(F')^{-4/3} = 9.7$$

$$(F')^{4/3} = \frac{200}{9.7} = 20.6$$

$$F' = 9.7$$

For $(y/B)' = 200$, $F' = 9.7$, Fig. 9 yields $S_o' = 20$.

Finally

$$\frac{S_o}{S_o'} = \left(\frac{F}{F'}\right)^{2/3} = 8.85$$

$$S_o = 177$$

The average plume dilution is then

$$S_a = \sqrt{2} \times 177 = 250$$

For the previous solution for 50 separate round jets it was found that $S_o = 115$ (centerline) and $S_a = 200$ (average). Thus, it is indicated that a closer approach to a line source by using more ports of smaller diameter and closer spacing could improve the dilution from about 200 up to a maximum of about 250; this presumes that the overall diffuser length is kept constant (50 x 25 = 1250 ft) and that the discharge velocity is not changed. To increase the dilution beyond 250 would require increasing the total diffuser length in order to reduce the unit discharge q (cf. plume formula, Eq. 26).

The validity of the lower right part of Fig. 9 (where the S_o contours slope down) is open to serious question. When the depth is small and the jetting strong along a line source, the limiting factor may be access for the water supposedly entrained. The dilution in this case may be governed by the overall flow-field dynamics, and is not well represented by jet mechanics based on the assumption of an infinitely deep flow field.

Slot Buoyant Jet in a Current (Uniform Environment)

Under this project, Cederwall (1971, A-6) studied the problem of a two-dimensional buoyant jet by laboratory experiments and dimensional analysis, including cases with and without ambient current. One of his principal objectives was to identify the important flow regimes, as shown by the next three figures (Figs. 10, 11, 12) taken from his report.

The flow regime for a buoyant slot jet discharging near the bottom is characterized by the following variables for cases of relatively small volume flux (following Cederwall's notation):

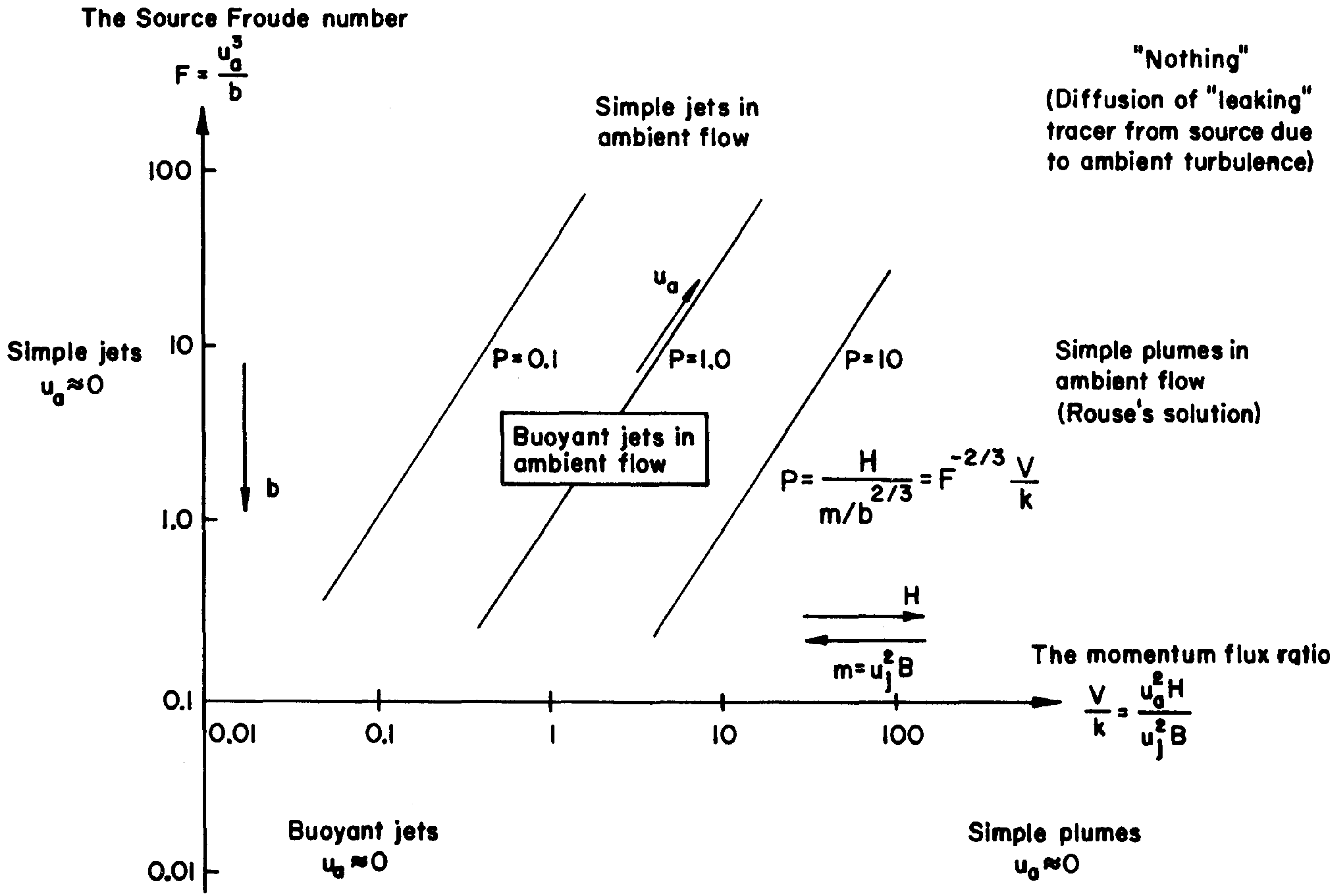


Figure 10. Flow regimes for a slot jet in a current (keeping θ = constant). The general flow situation in the center of the graph is surrounded by the limiting cases. The parameter P is the ratio between the flow depth H and a characteristic length $m/b^{2/3}$ of the source. (After Cederwall, 1971, A-6.)

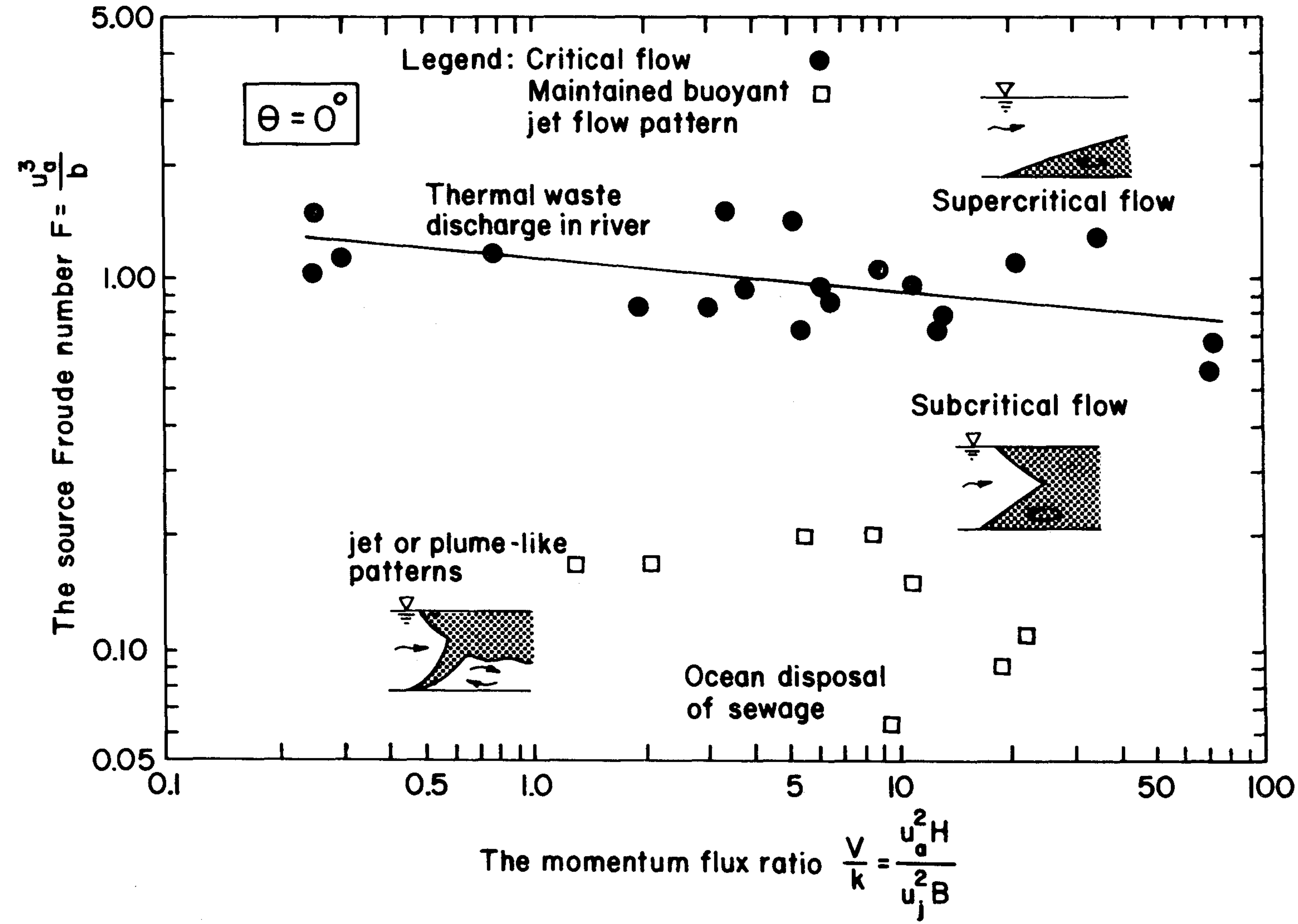


Figure 11. Observed flow regimes for a horizontal buoyant slot jet in a co-flowing stream. Critical flow is defined as the situation when the formation of a surface wedge is incipient at the reference section. (After Cederwall, 1971, A-6.)

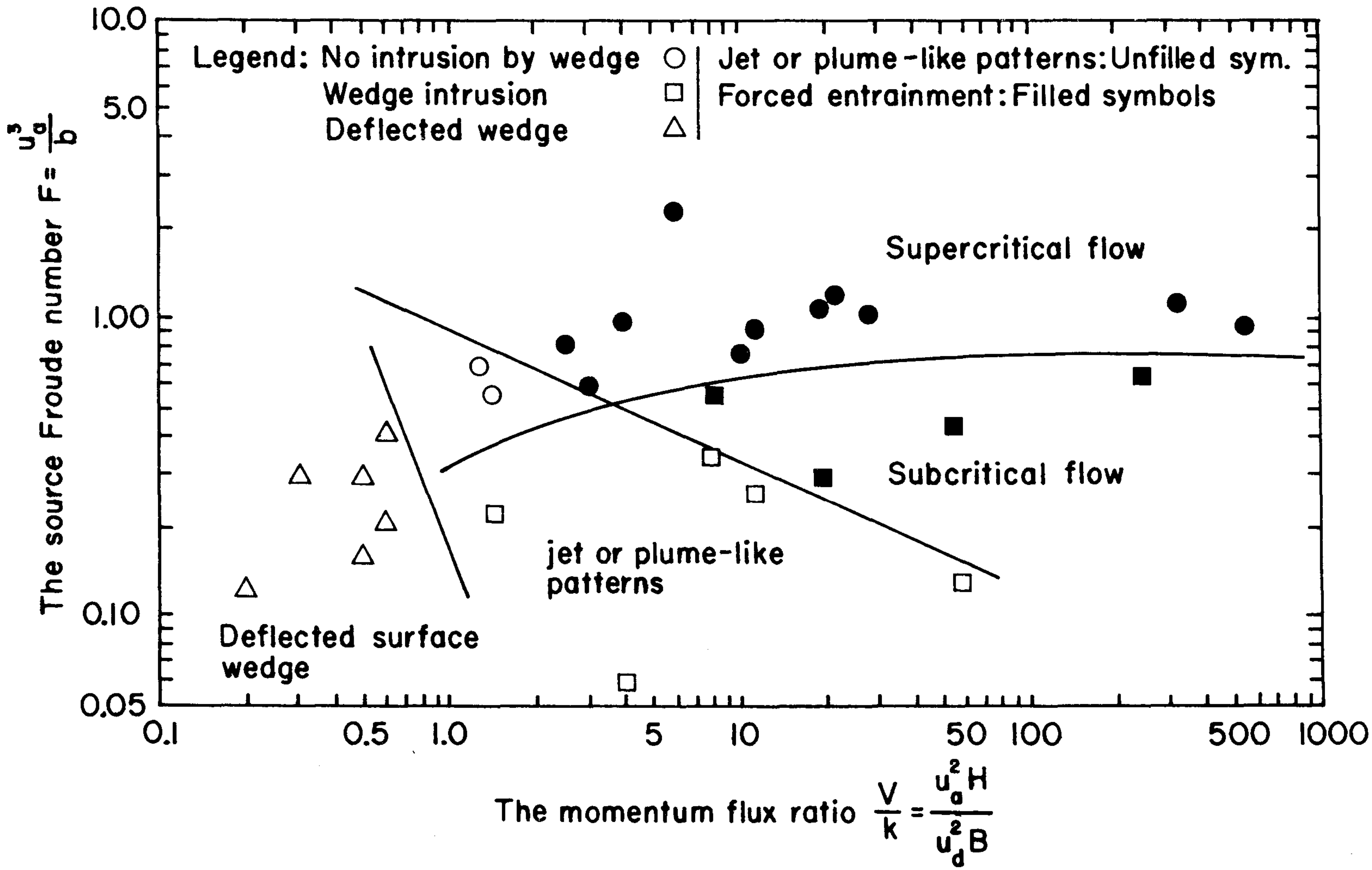


Figure 12. Observed flow regimes for a vertical buoyant slot jet in a cross stream. Forced entrainment is defined as a situation where the typical buoyant jet flow pattern breaks up and there is efficient mixing close to the source. (After Cederwall, 1971, A-6.)

$m = u_j^2 B$ = kinematic momentum flux of the source

$b = \frac{\rho_o - \rho_1}{\rho_o} g\, u_j B$ = kinematic buoyancy flux

θ = angle of injection ($\theta = 0$ is horizontal in downstream direction)

u_a = ambient current velocity

H = total depth of flow.

The flow pattern is just characterized by the following three dimensionless groups:

1. $F = \frac{u_a^3}{b}$ = Froude number (strength of current relative to buoyancy flux at source)
2. $\frac{u_a^2 H}{m} = \frac{u_a^2 H}{u_j^2 B}$ = momentum flux ratio (current/source)
3. θ

The source Froude number represents the strength of the ambient current relative to the buoyancy of the source: the greater F is, the more the plume will be "bent over" by the current. Similarly the ratio $u_a^2 H/u_j^2 B$ indicates how strong the current is relative to the source momentum.

For a given angle of injection θ, the flow regions can be classified as shown schematically in Fig. 10, with indication of various limiting cases. Arrows indicate which way one moves on the graph when certain variables are changed; for example, when the ambient velocity is increased, one moves along lines of 3/2 slope on a log-log graph. The extreme case in the upper right is for very small source buoyancy and momentum, in which case there is no dynamic effect of the source but only passive diffusion in the boundary layer due to ambient turbulence. Rouse's solution (4) is the limiting case of small buoyancy input and negligible source momentum.

In Fig. 11, the principal flow regimes for horizontal buoyant jets are defined on the basis of flume experiments; Fig. 12 gives the same for vertical jets. The purpose of these figures is to show from exploratory experiments the range and complexity of possible flow regimes. The solutions for slot buoyant jets obviously apply only in the lower left corners of Figs. 11 and 12 as the asymptote for $u_a \rightarrow 0$. It is not possible to give a definitive quantitive criterion at this time for how small u_a must be for Fig. 9 to apply, although Fig. 11 gives some guidance.

In the course of Cederwall's experiments it was found that these two-dimensional stratified flows are sensitive to boundary conditions at the end of the flume and the duration of an experiment. The diluting water on the downstream side of a buoyant slot jet in a current can only come from downstream because the rising jet itself (from wall to wall of the flume) prevents the cool water from upstream from getting underneath. However, in prototype conditions all line sources are finite, and therefore some flow of diluting water from the sides to the downstream side of the jet is always possible. In two-dimensional flume experiments it is not clear how the downstream condition should be modelled. Consequently, the results of Figs. 11 and 12 should be regarded as preliminary until there is further investigation of the sensitivity to downstream boundary conditions, and their relation to three-dimensional aspects of the far field.

In case of a light current and deep water, there may develop a thick pollutant field over a line diffuser. Although the dynamics of the rising plume may be little affected, nevertheless the height of rise through uncontaminated water is reduced. The entrainment of clean diluting water is reduced, and a correction to the initial dilution is required. An empirical procedure is given at the end of the next section for making this correction (p. 54).

SECTION VI

RESULTS FOR TURBULENT BUOYANT JETS IN STRATIFIED ENVIRONMENTS

This section summarizes important results for three- and two-dimensional buoyant jets into quiescent density-stratified environments. An approximate graphical method is also given for determining the maximum height of rise in environments with irregular density profiles. Exact solutions may be made by programs developed by Ditmars (1969, A-3) and Sotil (1971, A-11).

Round Buoyant Jet (Linearly Stratified Environment, No Current)

The basic flow pattern for this case is shown in Fig. 2 (lower half). It is necessary to assume a linear density profile in order to get generalized solutions. Non-linear or irregular profiles can often be approximated as linear with reasonably good results, or numerical solutions may be generated specially on a case-by-case basis.

The important characteristics of the solutions are the maximum height of rise y_{max} and the dilution of a tracer at the top of the rising plume (assuming constant background concentration of the tracer over depth). Figs. 13 and 14 from Fan and Brooks (1969, A-2) give the basic results in dimensionless form. The definitions are as follows:

θ = angle of discharge (0 = horizontal, 90^o = vertical in direction of buoyancy)

y_{max} = height of rise from source to top of plume

ξ_t = dimensionless height of rise $\propto y_{max}$

m_o = dimensionless momentum flux parameter at the end of the zone of flow establishment

μ_o = dimensionless volume flux parameter at the end of the zone of flow establishment

μ_t = dimensionless volume flux parameter at the top of plume

S_t = centerline dilution at the top of the plume (relative to end of zone of flow establishment)

$= \mu_t/\mu_o$

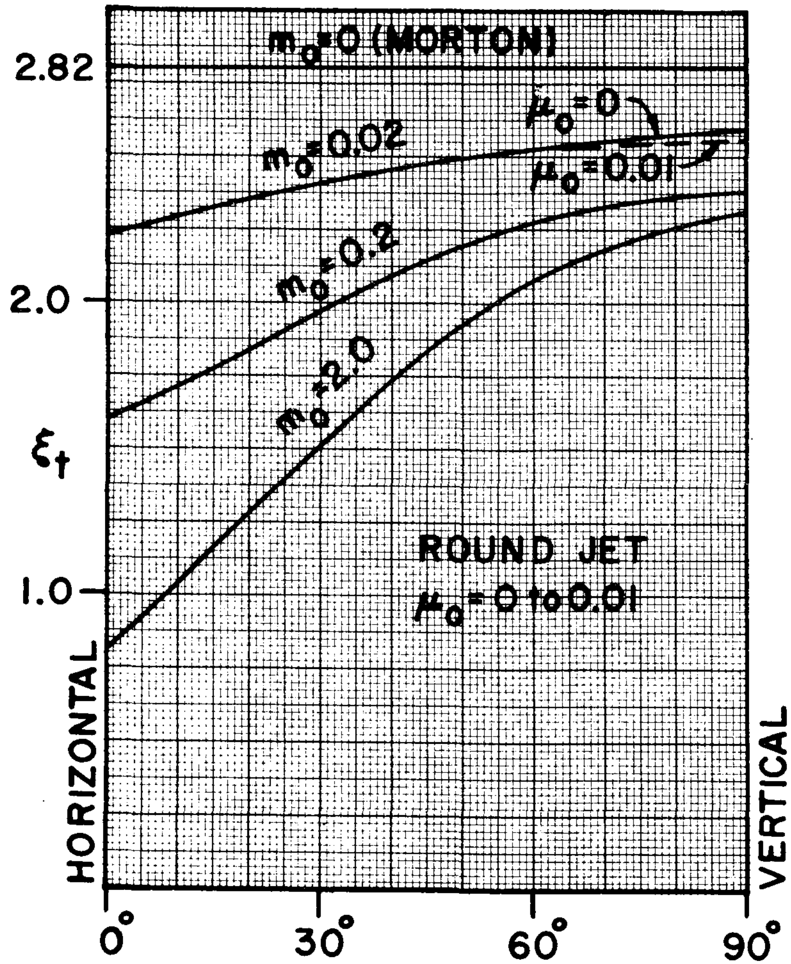


Figure 13. Terminal height of rise ξ_t for inclined round buoyant jets with μ_o = 0 to 0.01. (After Fan and Brooks, 1969, A-2.)

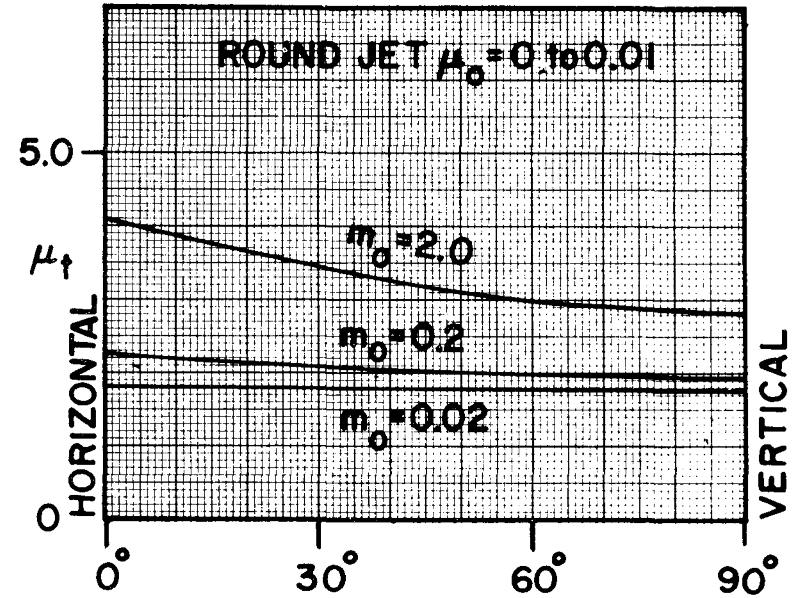


Figure 14. Terminal volume flux parameter μ_t for inclined round buoyant jets with μ_o = 0 to 0.01. (After Fan and Brooks, 1969, A-2.)

The variables may be related to two other variables F and T as follows:

$$F = \frac{U_o}{\sqrt{\frac{\rho_o - \rho_1}{\rho_o} gD}} = \text{densimetric Froude number} \tag{29}$$

$$T = \frac{\rho_o - \rho_1}{D(-\frac{d\rho_a}{dy})} = \text{a stratification parameter} \tag{30}$$

where ρ_1 is the centerline density at the end of the zone of flow establishment, ρ_o is the reference density (ambient fluid at source level), and ρ_a is the ambient density. A small adjustment (-13%) is necessary to convert the discharge values of $(\rho_o-\rho_d)$ to $(\rho_o-\rho_1)$, as given by Eq. 14. With the entrainment coefficient α = 0.082 and the spreading ratio λ = 1.16:

$$m_o = 0.324\ F^2\ T^{-1} \tag{31}$$

$$\mu_o = 2.38\ F^{1/4} T^{-5/8} \tag{32}$$

$$S_{td} = 1.15\ \mu_t/\mu_o \quad \text{(centerline, relative to discharge)} \tag{33}$$

$$S_{ad} = 2\ \mu_t/\mu_o \quad \text{(average, relative to discharge)} \tag{34}$$

$$\frac{y_{max}}{D} = 1.37\ \xi_t\ F^{1/4}\ T^{3/8} \tag{35}$$

ξ_t and μ_t are solutions read from Figs. 13 and 14, or from more detailed figures in Fan and Brooks (1969, A-2, Ch. IV).

For most practical cases it is sufficient to use the plume solution ($m_o \sim 0$, $\mu_o \sim 0$); this will overestimate the height of rise slightly and underestimate the dilution. Therefore we may simplify Eq. 35 by putting in ξ_t = 2.82, and rearranging:

$$\frac{y_{max}}{D} = 3.86\ F^{1/4} T^{3/8} \tag{36}$$

or

$$y_{max} = 3.98\ (Qg'_d)^{1/4} (-\frac{g}{\rho_1} \frac{d\rho_a}{dy})^{-3/8} \tag{37}$$

where $Q = \pi U_o D^2/4$, and $g'_d = \frac{\rho_1 - \rho_d}{\rho_1} g$. For this case a better value of α is 0.093 according to Morton, Taylor and Turner (1); with this value the coefficient above in Eq. 36 becomes 3.75. The difference (less than 6%) is well within the limits of accuracy.

The centerline dilution for the limiting plume case is found by putting Eq. 32 and $\mu_t = 1.71$ (solution for $m_o = 0$, $\mu_o = 0$) into Eq. 33:

$$S_{td} = 1.15\ \mu_t/\mu_o = 1.15\ (1.71)(2.38)^{-1} F^{-1/4} T^{5/8}$$

$$S_{td} = 0.83\ F^{-1/4} T^{5/8} \tag{38}$$

or

$$S_{td} = 0.70\ (Q g'_d)^{3/4} Q^{-1} \left(- \frac{g}{\rho_1} \frac{d\rho_a}{dy}\right)^{-5/8} \tag{38a}$$

For another useful form, combine Eq. 38 with Eq. 36:

$$\frac{y_{max}}{D} S_{td} = 0.826\ (3.86) T = 3.19 T$$

$$y_{max}\ S_{td} = 3.19\ \frac{\rho_o - \rho_1}{- \frac{d\rho_a}{dy}}$$

$$S_{td} = 2.77\ \frac{\rho_o - \rho_d}{- \frac{d\rho_a}{dy}} (y_{max})^{-1}$$

Now for a linear profile we may define

$$\Delta\rho_a = - \frac{d\rho_a}{dy} y_{max} = \text{decrease in ambient density over height of rise } y_{max}$$

Therefore:

$$S_{td} = 2.8\ \frac{\rho_o - \rho_d}{\Delta\rho_a}, \tag{39}$$

and the average over the plume at the top is by Eq. 34:

$$S_{ad} = 4.8\ \frac{\rho_o - \rho_d}{\Delta\rho_a} \tag{40}$$

Another useful expression for the dilution at the top of the plume is obtained by eliminating the density gradient (or T) in combining Eq. 36 and 38.

$$S_{td} = 0.826\ F^{-2/3}\left(\frac{y_{max}}{D}\right)^{5/3}\ (3.86)^{-5/3}$$

$$= 0.0872\ \left(g\frac{\rho_o-\rho_1}{\rho_o}\right)^{1/3}\ y_{max}^{5/3}(U_oD^2)^{-2/3}$$

$$= 0.0872\ \left(\frac{1}{1.15}\ g_d'\right)^{1/3} y_{max}^{5/3}\ \left(\frac{4Q}{\pi}\right)^{-2/3}$$

$$S_{td} = 0.071\ g_d'^{1/3} y_{max}^{5/3}\ Q^{-2/3} \tag{41}$$

It is interesting to compare this result with Eq. 18 for non-stratified environment with height of rise y; the form of the equation is the same, but the coefficient 0.089 for the non-stratified case is 25% more than the coefficient (0.071) in Eq. 41. The reason for the difference is not readily apparent, but intuitively it may be attributed to the lower velocity of the rising plume as it approaches its terminal height in a stratified environment compared with the non-stratified case in which the flow reaches the level y (or surface) with appreciable residual velocity.

Example. Consider a discharge of 50 mgd (78 cfs) from a multiple-port diffuser at 100 ft depth when there is a density gradient in the ocean as follows:

Seawater, bottom (-100 ft), $\rho_a = \rho_1 = 1.02580$
Seawater, top $\rho_a = 1.02460$

$$-\frac{1}{\rho_1}\frac{d\rho_a}{dy} = \frac{.00120}{1.026(100)} = 1.17\text{x}10^{-5}\ \text{ft}^{-1}$$

(This density difference is approximately equivalent to a temperature difference of 6°C.) The density of discharge is $\rho_d = 0.99950$, and $g_d' = g(\rho_1-\rho_d)/\rho_1 = .0256g = 0.826$ ft/sec^2.

Determine whether the sewage field will be submerged or not for the case of (a) 50 ports and (b) 5 ports. For approximate analysis on the safe side, use the buoyant plume approximation and neglect the horizontal momentum and extra mixing induced near the bottom. Assume no interference between jets.

By Eq. 37, with Q per port = 78/50 cfs/port:

$$y_{max} = 3.98 \left(\frac{78}{50} \times 0.826\right)^{1/4} (32.2 \times 1.16 \times 10^{-5})^{-3/8}$$

$$y_{max} = 82 \text{ ft}$$

The centerline dilution is found by Eq. 39 to be

$$S_{td} = 2.8 \ \frac{.0263}{.0012 \times (82/100)} = 75.$$

The average is by Eq. 40:

$$S_{ad} = 128$$

For part (b), 5 ports, we obtain by ratios

$$Q = 15.5 \text{ cfs/port}$$

$$y_{max} = 82 \text{ ft} \left(\frac{15.5}{1.55}\right)^{1/4} = 146 \text{ ft.}$$

This result exceeds the total depth, and therefore the plume rises to the surface (y = 100 ft) and is not kept submerged by the stratification. From this example, it is apparent that multiple jet diffusers greatly enhance the possibility of generating a submerged sewage field.

Slot Buoyant Jet (Linearly Stratified Environment, No Current)

The basic flow pattern for this case is shown in Fig. 3 (lower half). The development for slot buoyant jets parallels that for round buoyant jets described in the preceding section. The basic results from Fan and Brooks (1969, A-2) are shown in Figs. 15 and 16. The definitions are as follows:

$$B = \text{slot jet width or equivalent } \left(= \frac{\pi}{4}\frac{D^2}{s}\right)$$

$$F = \frac{U_o}{\sqrt{\frac{\rho_o - \rho_1}{\rho_o} gB}} = \text{densimetric Froude number} \tag{42}$$

$$T = \frac{\rho_o - \rho_1}{B\left(-\frac{d\rho_a}{dy}\right)} = \text{stratification parameter} \tag{43}$$

Assuming $\alpha = 0.14$ and $\lambda = 1.00$ (slightly changed from Fan and Brooks), the solution parameters are:

$$m_o = 0.500\ F^2\ T^{-1} \tag{44}$$

$$\mu_o = 1.85\ F^{2/3} T^{-1} \tag{45}$$

$$\frac{y_{max}}{B} = 0.96\ \xi_t F^{1/3} T^{1/2} \tag{46}$$

The dilution at the top of the plume S_t needs no correction for the zone of flow establishment because $\lambda = 1.00$ ($\rho_1 = \rho_d$):

$$S_t = \sqrt{\mu_t/\mu_o} \qquad \text{(centerline)} \tag{47}$$

$$S_a = \sqrt{2\mu_t/\mu_o} \qquad \text{(average)} \tag{48}$$

The values of ξ_t and μ_t are the solutions read from Figs. 15 and 16, or from more detailed figures in Fan and Brooks (1969, A-2, Ch. VI).

For most practical cases it is sufficient to use the plume solution ($m_o \sim 0$, $\mu_o \sim 0$); this will overestimate the height of rise slightly and

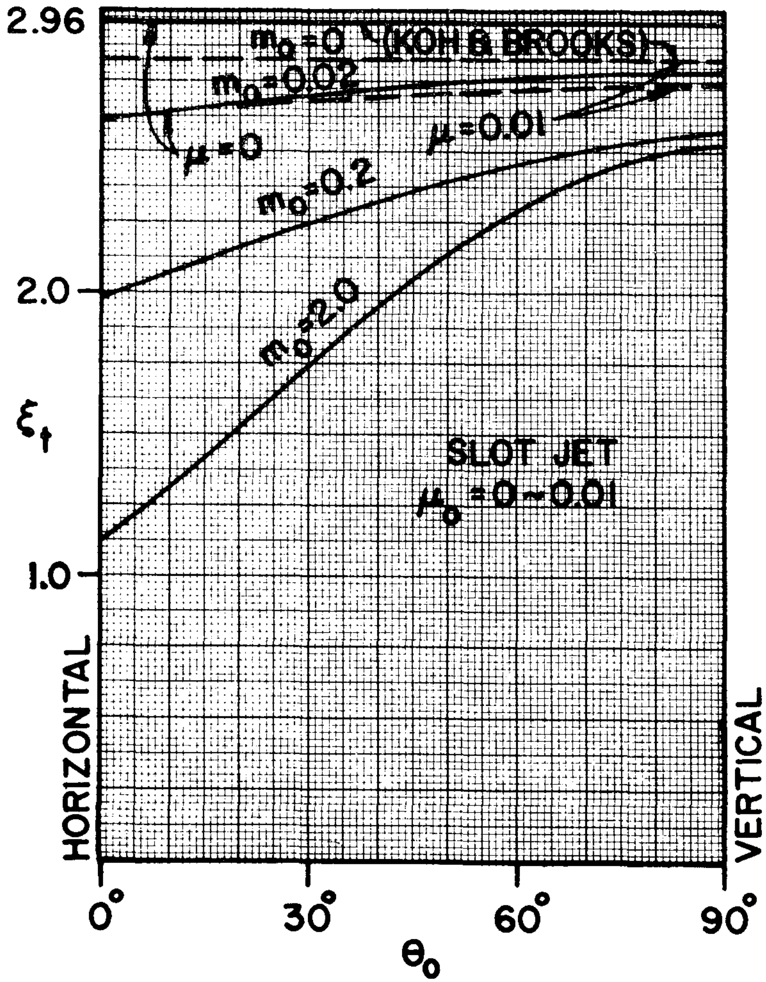


Figure 15. Terminal height of rise ξ_t for inclined slot buoyant jets with μ_o = 0 to 0.01. (After Fan and Brooks, 1969, A-2.)

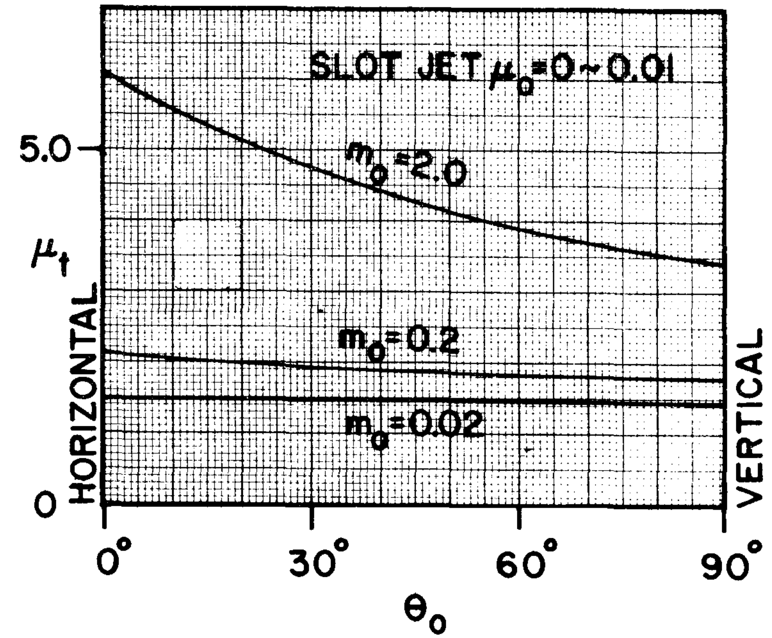


Figure 16. Terminal volume flux parameter μ_t for inclined slot buoyant jets with μ_o = 0 to 0.01. (After Fan and Brooks, 1969, A-2.)

underestimate the dilution. By putting $\xi_t = 2.96$ in Eq. 46, and rearranging

$$\frac{y_{max}}{B} = 2.84\ F^{1/3}T^{1/2} \tag{49}$$

$$y_{max} = 2.84\ (qg')^{1/3}\left(-\frac{g}{\rho_1}\frac{d\rho_a}{dy}\right)^{-1/2} \tag{50}$$

wherein $g' = \frac{\rho_1 - \rho_d}{\rho_1} g$ and q = discharge per unit length = U_oB. The final coefficient in this case has not been directly confirmed by experiments; the above result is entirely theoretical, based on α and λ for buoyant plumes in a non-stratified fluid. No experiments were made for this case during this project.

The centerline dilution for the limiting plume case is obtained from the solution $\mu_t = 1.41$ (for $m_o \sim 0$, $\mu_o \sim 0$) and the substitution of Eq. 45 into Eq. 47:

$$S_t = \sqrt{\frac{1.41}{1.85}}\ F^{-1/3}\ T^{1/2}$$

$$S_t = 0.87\ F^{-1/3}\ T^{1/2} \tag{51}$$

$$S_t = 0.87\ (qg')^{2/3}q^{-1}\left(-\frac{g}{\rho_1}\frac{d\rho_a}{dy}\right)^{-1/2} \tag{52}$$

For the average dilution multiply Eq. 52 by $\sqrt{2}$.

In terms of $\Delta\rho$ over the height of rise, defined as

$$\Delta\rho = -\ y_{max}\frac{d\rho_a}{dy}\ ,$$

we obtain from Eqs. 49 and 51:

$$\frac{y_{max}}{B}\cdot S_t = 2.84(0.87)T$$

$$S_t = 2.5\ \frac{BT}{y_{max}}$$

$$S_t = 2.5\ \frac{\rho_1 - \rho_d}{\Delta\rho_a}\ , \tag{53}$$

and the average dilution by Eq. 48 is

$$S_a = 3.5 \frac{\rho_1 - \rho_d}{\Delta\rho_a} \tag{54}$$

The centerline dilution at the top is also conveniently expressed in terms of y_{max} by eliminating the density gradient (or T) from Eq. 49 and 51:

$$\frac{S_t B}{y_{max}} = \frac{0.87}{2.84} F^{-2/3}$$

$$S_t = 0.31 \, g'^{1/3} y_{max} q^{-2/3} \tag{55}$$

This may be compared with the corresponding line plume result for the non-stratified case, Eq. 26, which is of the same form but with coefficient of 0.38 instead of 0.31, and y replacing y_{max}. The coefficient is 23% larger for the non-stratified case; a nearly similar percentage difference (25%) was found for the round plume dilution formulas (non-stratified vs. stratified). Again the difference might be attributed to the slower rate of rise in the stratified case because the plume reaches the top at zero velocity compared to "bumping" the free surface with significant residual velocity. (The basic entrainment equation presumes that entrainment is proportional to plume velocity, Eq. 5.)

The formulas for height of rise (Eq. 50) and for dilution (Eq. 52, 53, 54 and 55) are for the limiting case of a buoyant plume, and are thus conservative for design; in other words, the height of rise is overestimated and the dilution underestimated. More realistic coefficients in the equations may be found by calculating m_o and μ_o for a typical line diffuser, and determining values of ξ_t and μ_t from Figs. 15 and 16. Representative values for a line diffuser for sewage discharge in the ocean might be as follows:

U_o = discharge velocity = 10 ft/sec

q = discharge per unit length = 0.07 ft^2/sec

$$B = q/U_o = 0.007 \text{ ft}$$

$$\Delta\rho = \rho_o - \rho_d = 0.026$$

$$-\frac{d\rho_a}{dy} = 0.001/100 \text{ ft} = 10^{-5}\text{ft}^{-1}$$

$$\rho_o = 1.025$$

$$g' = \frac{\Delta\rho}{\rho_o} g = 0.84 \text{ ft/sec}^2$$

$$F = \frac{U_o}{\sqrt{g'B}} = \frac{10}{\sqrt{.84 \times 0.007}} = 131$$

$$T = \frac{\rho_o - \rho_d}{B(-\frac{d\rho_a}{dy})} = \frac{0.026}{0.007 \times 10^{-5}} = 3.7 \times 10^5$$

$$m_o = 0.50 \ F^2 T^{-1} = 0.023 \qquad \text{(Eq. 44)}$$

$$\mu_o = 1.85 \ F^{2/3} T^{-1} = 1.3 \times 10^{-4} \qquad \text{(Eq. 45)}$$

The purpose of the above calculation is only to establish orders of magnitude; we find that μ_o is so small ($\sim 10^{-4}$) that it has negligible effect on the plume dynamics, while the magnitude of m_o (~ 0.02) does affect the solution slightly. We find from Figs. 15 and 16 (or Ref. A-2):

	Buoyant slot jet ($m_o \sim 0.02$)		
	Vertical discharge	Horizontal discharge	Plume values
ξ_t	2.78	2.61	2.96
μ_t	1.46	1.48	1.41

Often a horizontal discharge is used, but with the flow split between the two sides of the pipe (ports on both sides). The initial momentum theoretically cancels out for the whole flow pattern, but there is still the benefit of high dilution with "heavy" bottom water. In this case, it is recommended that the flow be regarded as a single line, but that credit be allowed for the momentum by using $\xi_t = 2.61$ and $\mu_t = 1.48$ for horizontal discharge. In other words we are saying that the dilution obtained with ports on both sides is presumed to be as good as it would be if they were all on one side, jetting horizontally.

The revised equations, recommended for design purposes for line multiport diffusers in a stratified environment are as follows:

Height of rise:

$$y_{max} = 2.5 \ (qg')^{1/3} \left(- \frac{g}{\rho_1} \frac{d\rho_a}{dy}\right)^{-1/2} \tag{56}$$

Centerline dilution at top:

$$S_t = 0.89 \ (qg')^{2/3} \ q^{-1} \left(- \frac{g}{\rho_1} \frac{d\rho_a}{dy}\right)^{-1/2} \tag{57}$$

$$S_t = 2.2 \ \frac{\rho_1 - \rho_d}{\Delta\rho_a} \tag{58}$$

$$S_t = 0.36 \ g'^{1/3} \ y_{max} \ q^{-2/3} \tag{59}$$

For average dilution, multiply above values by $\sqrt{2}$.

The numerical coefficients given in the formulas above differ somewhat from those given previously (Brooks, 1970, B-1; Fischer and Brooks, 1970, B-2); the values of α and λ have been adjusted and some arithmetic errors corrected.

Example. For the data given immediately above we may make the calculation of y_{max} and S_t from Equations 56 and 59 respectively, as follows:

$$y_{max} = 2.5 \ (0.07 \times 0.84)^{1/3} \left(- \frac{32.2}{1.025} \ 10^{-5}\right)^{-1/2}$$

$$y_{max} = 55 \text{ ft.}$$

$$S_t = 0.36 \ (0.84)^{1/3} \ (55) \ (0.07)^{-2/3}$$

$$S_t = 110 \text{ (centerline)}$$

$$S_a = 155 \text{ (average)}$$

Approximate Solutions for Buoyant Jets in Environments with Non-Linear Stratification

In practice, the ambient density stratification will never be exactly linear; to solve buoyant jet problems one may either make a direct computer solution, or resort to a linear approximation of the measured profile, using the formulas of the preceding subsections.

For direct computer solutions, the reader is referred to programs developed for this project: for the round buoyant jet, Ditmars (1969, A-3) and for the slot buoyant jet, Sotil (1971, A-11). The program input for each includes the measured density profile for the environment, represented by the density values at selected elevations in the fluid. The solution proceeds from the same assumptions and equations as used for the cases of constant density gradients.

For most problems, a reasonable approximate solution (Brooks, 1970, B-1) is obtained by assuming an equivalent uniform density gradient over that part of the total depth through which the buoyant plume rises. In Fig. 17 the point of discharge is at 0 (neglect the size of the zone of flow establishment), with y measured upward from this point. The measured density profile is plotted from the water surface down, and the point 0 may be shifted up or down along the profile in the process of testing different designs. Between 0 and any point A the mean gradient is

$$- \frac{d\rho}{dy}\bigg|_{ave} = \frac{\Delta\rho_a}{y_a} \tag{60}$$

For simplicity $\Delta\rho$ is taken as positive for a stable profile, thus making the $\Delta\rho$-axis read backwards (measuring from right to left). To find the height of rise, y_{max}, let the point A be at the level $y_a = y_{max}$, and substitute Eq. 60 into Eq. 37 for round buoyant jets:

$$y_{max} = 3.98 \, (Qg_d')^{1/4} \left(\frac{g}{\rho_1} \frac{\Delta\rho_a}{y_{max}}\right)^{-3/8} \tag{61}$$

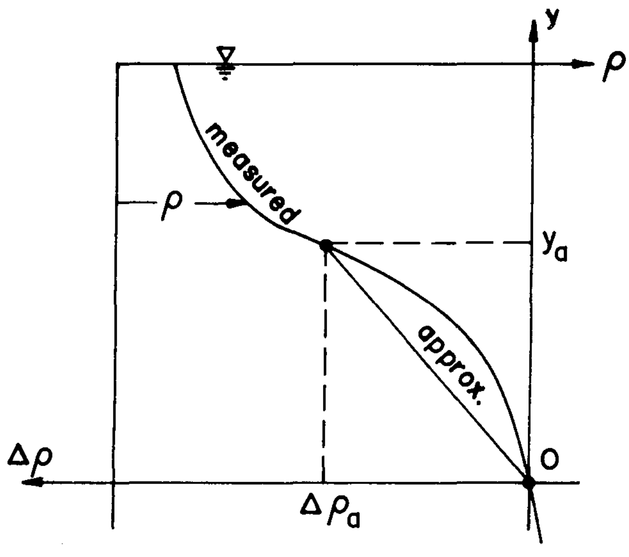


Figure 17. Approximation of non-linear density profile by a linear one.

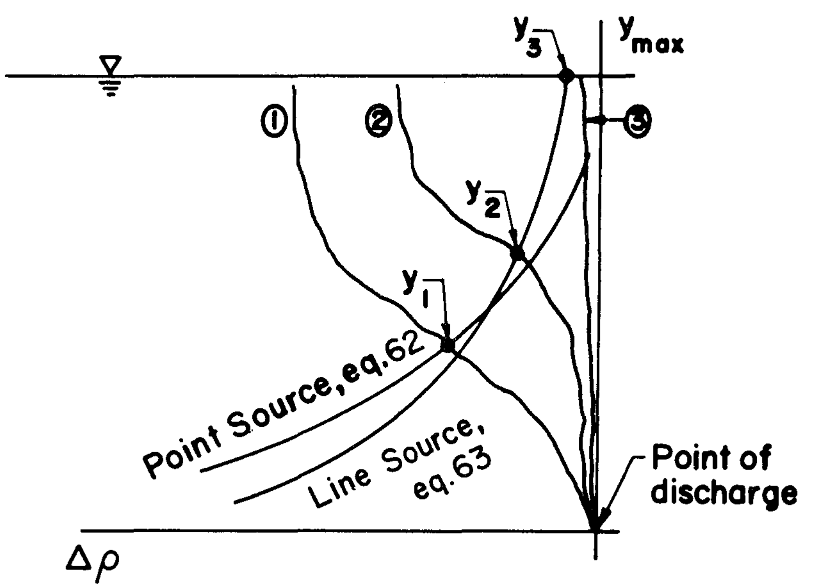


Figure 18. Diagram for solving Equations 62 and 63 with measured density profiles. (After Brooks, 1970, B-1.)

Simplifying:

$$y_{max} = 9.1(Qg_d')^{2/5} (g \frac{\Delta\rho_a}{\rho_1})^{-3/5} \tag{62}$$

Similarly for <u>line</u> buoyant jets, Eqs. 56 and 60 yield

$$y_{max} = 6.25 \ (qg')^{2/3} \ (g \frac{\Delta\rho_a}{\rho_1})^{-1} \tag{63}$$

Equations 62 and 63 may now be plotted on a graph of y_{max} vs. $\Delta\rho$; each represents all possible solutions $(y_{max}, \Delta\rho_a)$ for the given buoyancy flux of the source (point or line). If these curves are placed on a transparent graph overlying measured profiles, as shown in Fig. 18, the solutions are represented by the intersections (i.e. we find the solution values of $(y_{max}, \Delta\rho_a)$ which satisfy the measured profiles).

Note that for high values of $\Delta\rho$ the point source solution gives higher values of y_{max}, whereas for small $\Delta\rho$ the line source solution gives the higher values of y_{max}. It is recommended that in either case the <u>higher</u> value of the solution y_{max} always be used as a best estimate. For large heights of rise, the flow pattern over a multiple-port diffuser will become essentially a <u>line</u> plume (with interference between jets in a row), whereas for low rises the diffuser may generate essentially a series of independent round plumes, which stop rising before significant overlapping occurs. For the three ambient profiles in Fig. 18, the solutions are shown as y_1, y_2, y_3. Note that profile 3 allows the plume to reach the surface. With a transparency for the curves representing Equations 62 and 63, it is easy to investigate different possible depths of discharge at many times of year (varying density profiles).

This procedure can not be expected to be highly accurate, but nonetheless, it clarifies the relationships between the variables, and provides a rational basis for selecting a diffuser length and depth for achieving submergence various percentages of the time.

The dilutions are now readily obtained by applying Eq. 39 for round buoyant plumes; or Eqs. 53 or 58 for buoyant slot plumes or jets. For a given effluent density, the dilution is simply proportional to $(\Delta\rho_a)^{-1}$. Alternately the dilutions may be determined from y_{max} values by Eq. 41 for round plumes, Eq. 55 for slot plumes, or Eq. 59 for typical slot jets. It is important to note that when the density gradient is strong the dilution is decreased because y_{max} is less. The desire for submergence must be balanced with the need for high dilutions in developing an optimum design for an outfall diffuser.

This procedure has been applied successfully to several outfall designs, the first of which was a 27,400-foot long outfall, 10 feet in diameter, with a 6,000-foot long diffuser, built for the Orange County Sanitation District, California (5).

Reduction of Dilution by Blockage of Sewage Field for Line Diffuser

The dilution formulas may substantially over-estimate the dilution over line diffusers in case of light currents. In this case the sewage field may grow to such thickness over the diffuser that the height of rise through uncontaminated water may be reduced. Thus the entrainment of new diluting water is reduced, while continued plume rise within the sewage field itself does not increase the net dilution. The internal dynamics remain about the same.

The following approximate analysis may be used for cases with or without ambient stratification (see Fig. 19). As an approximation the "fallback" from the top of the plume to the top of the field is neglected.

Let b = average width of cloud

L = diffuser length

y_{max} = maximum height of rise above diffuser (may be taken as total depth if unstratified)

h = thickness of sewage field

U = current speed

$y = y_{max} - h$ = height from diffuser to bottom of sewage field

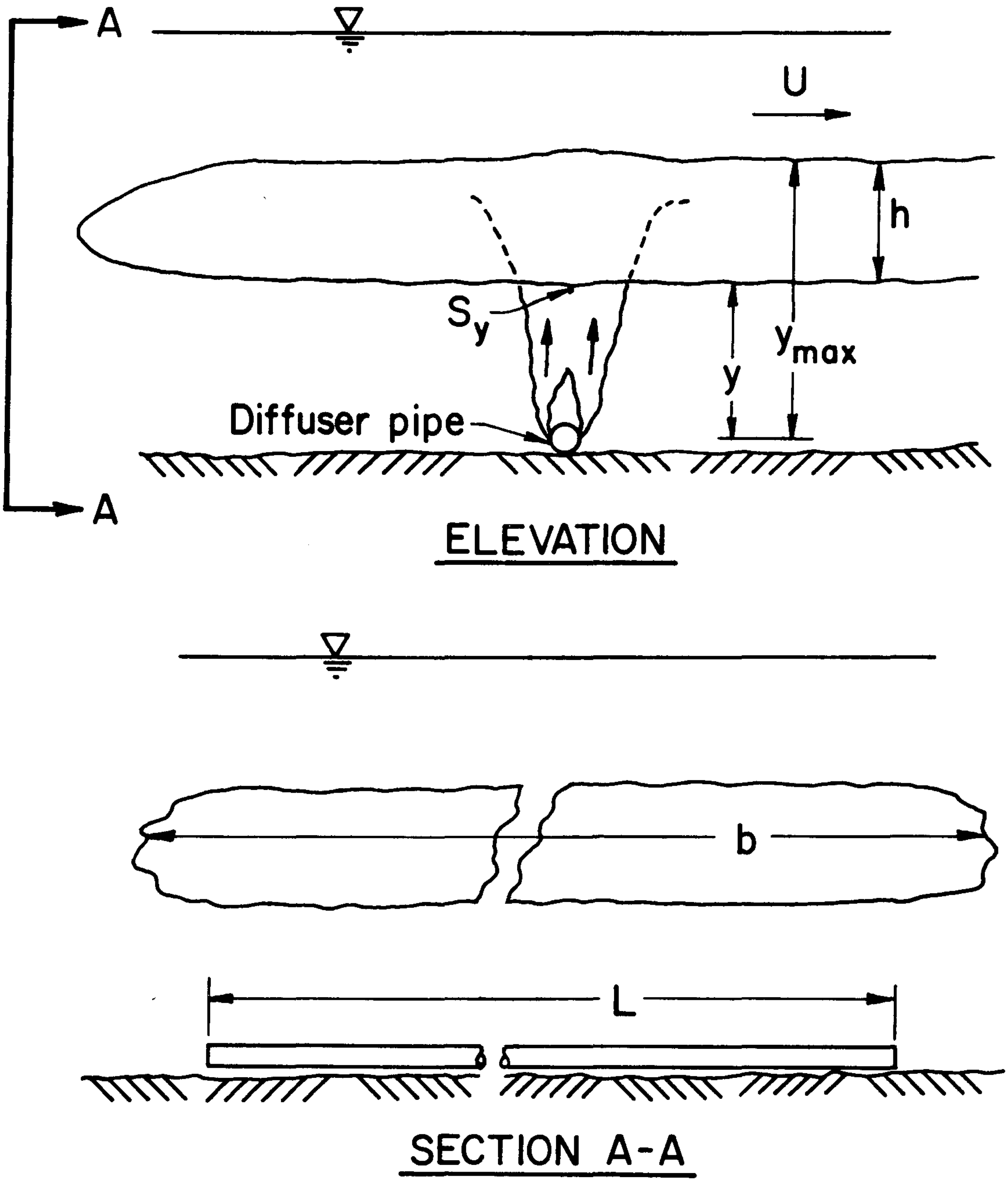


Figure 19. Schematic diagram of blocking of part of the water column by the pollutant field.

Q_s = sewage discharge

S_t = centerline dilution at top of plume as calculated by formulas assuming no blockage

S_y = centerline dilution at height y (bottom of sewage field) which is taken to be the reduced dilution for the plume because of blockage

S_a = average dilution in sewage field over diffuser

For all buoyant plumes (with or without stratification), the dilution is approximately linear with height, that is:

$$\frac{S_y}{S_t} = \frac{y}{y_{max}} \tag{64}$$

The average dilution in the field is $\sqrt{2}$ times the centerline value, or

$$S_a = \sqrt{2}\, S_y \ . \tag{65}$$

But the efflux from the outfall site may be expressed by the continuity equation

$$S_a Q_s = Ubh = Ub(y_{max} - y) \tag{66}$$

Hence,

$$S_y = \frac{Ub(y_{max} - y)}{\sqrt{2}\ Q_s} \tag{67}$$

Dividing by S_t

$$\frac{S_y}{S_t} = \frac{Uby_{max}}{\sqrt{2}\ Q_s S_t} \left(1 - \frac{y}{y_{max}}\right) \tag{68}$$

Let P be a factor defined as

$$P = \frac{\sqrt{2}\ Q_s S_t}{Uby_{max}} \tag{69}$$

and using Eqs. 64 and 69 in 68, we get

$$\frac{S_y}{S_t} = \frac{1}{P} \left(1 - \frac{S_y}{S_t}\right) \tag{70}$$

or

$$S_y = S_t \left(\frac{1}{1+P}\right) \tag{71}$$

This is the desired result. One computes the dilution S_t without considering blockage, and then finds the correction factor $\frac{1}{1+P}$ for blockage based on known quantities. The average field dilution is finally given by Eq. 65.

If P = 2 the analysis indicates that the sewage cloud has filled the upper 2/3 of the space with only 1/3 left at the bottom for entry of new water. For $P \gtrsim 2$, the above analysis is not to be trusted, and the whole dilution phenomenon is probably limited by far-field phenomena rather than near-field (i.e. diffuser details). This appears to be especially so for thermal outfalls, and additional research is underway.

Final Comment

The results of calculations for buoyant jets in a stratified environment are probably no more accurate than ± 20%. The results are primarily theoretical, with quantitative confirmation only for round buoyant jets in laboratory tanks. Field data check these results qualitatively for outfall diffusers recently built, but this writer knows of no quantitative studies of submerged sewage fields which are sufficiently detailed to evaluate the formulas above. At the field scale, measurements are extremely difficult; however, more laboratory work on line buoyant jets is badly needed.

SECTION VII

RELEASE OF A SLUG OF DENSE FLUID INTO A TWO-LAYERED ENVIRONMENT

Problem

When a slug of sludge or dredge solids is dumped into the sea or any other body of water, it may sink only to the level of a density interface such as a thermocline. Although the initial slug is heavier than the upper layer and starts sinking, the entrainment of the upper layer may so reduce the effective density of the slug that it does not penetrate into the lower more dense layer. Sullivan (1972, A-8) investigated this problem with dimensional analysis and small-scale laboratory experiments. First he considered the case of a slug without initial momentum, and then with initial momentum. In the latter case, distinct vortex rings formed. This problem is similar to the idea of puffing smoke out of a smokestack in an attempt to penetrate atmospheric inversion layers.

Results

Only the case of zero initial momentum will be considered here. The important variables are defined as follows (see Fig. 20):

V = volume of heavy fluid initially released

ρ_i = initial density of heavy fluid injected

ρ_1 = density of upper layer

ρ_2 = density of lower layer

g = gravity

Z = depth from release point to interface

Under the assumption that all the density differences are small compared to ρ, the variables of physical significance are:

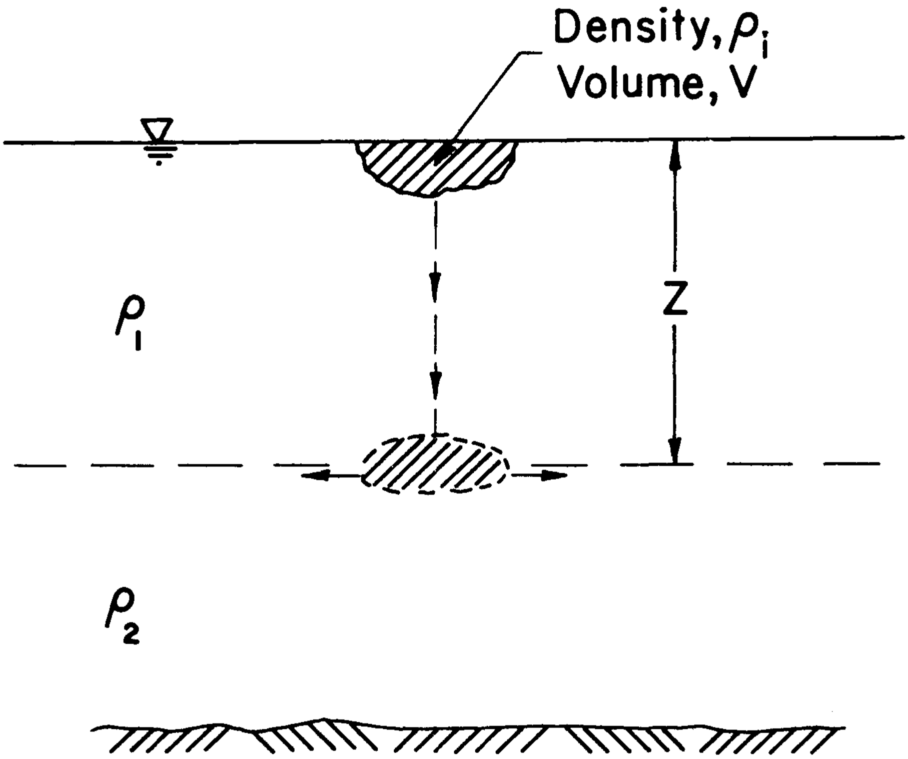


Figure 20. Definition sketch for release of heavy fluid in a two-layer environment.

$g(\rho_i - \rho_1)V$ = submerged weight (the driving force of the flow pattern)

Z = distance of travel to the interface

$g(\rho_2 - \rho_1)$ = difference in unit weight between the layers

These three variables may be combined into just one dimensionless group which must characterize the flow pattern:

$$\Lambda = \frac{(\rho_2 - \rho_1)\ Z^3}{(\rho_i - \rho_1)\ V} \tag{72}$$

Sullivan found the following criteria experimentally:

$\Lambda > 29$	Less than 10% of injected slug penetrates the lower layer
$\Lambda < 1.5$	More than 90% of injected slug continues into the lower layer

These numbers should be considered tentative because the experiments were conducted at low Reynolds number for which the flow was partly laminar.

<u>Example</u>. A load of 1,000 ft^3 of sludge at $\rho = 1.0300$ is to be suddenly dumped in the ocean ($\rho_1 = 1.0245$); at 40 feet depth there is a thermocline below which $\rho_2 = 1.0255$. Find whether the sludge sinks into the lower layer.

Compute Λ by Eq. 72:

$$\Lambda = \frac{(1.0255 - 1.0245)40^3}{(1.0300 - 1.0245)1000}$$

$$= 11.6$$

As this number is below 29 but above 1.5, there will be substantial but not complete penetration of the thermocline by the sinking fluid mass. The outer portions of the sinking mass will entrain enough of the fluid of the upper layer to make the mixture density less than that of the lower layer.

If the sludge is discharged with some initial velocity (for instance, if dropped on the water surface) then the penetration would be greater. (See Sullivan, 1972, A-8 for details of analysis.) On the other hand, if the volume V were ten times smaller, then it would not penetrate the interface. A slow continuous release would also have less penetration. Of course, whether penetration of the thermocline is desirable or not depends on the circumstances of a particular problem.

SECTION VIII

SELECTIVE WITHDRAWAL AND ARTIFICIAL MIXING IN DENSITY-STRATIFIED RESERVOIRS

Selective Withdrawal

Another important stratified flow pattern occurs at the outlet of a dam in a stratified water reservoir as depicted in Fig. 21. A withdrawal layer forms at the level of the outlet because lighter fluid above and heavier fluid below cannot be drawn through the outlet.

Brooks and Koh (1969, C-1) have provided a detailed review of this problem and have recommended an analysis for extending the earlier work of Koh (6) to apply to large-scale turbulent reservoirs. A linear approximation is used for the ambient density profile, and the reservoir flow pattern is considered to be two-dimensional (i.e. uniform across the reservoir). The remainder of this sub-section is extracted from Brooks and Koh (1969, p. 1396-7).

For turbulent withdrawal flows away from immediate vicinity of the outlet, it is hypothesized that Koh's viscous diffusive experiments and analysis can be applied by replacing ν and D by E_m, the vertical eddy diffusivity. For self-generated turbulence, it is predicted that the proper characteristic length is $a = (q/\sqrt{g\varepsilon})^{1/2}$, where q is the unit discharge and $\varepsilon = -\frac{1}{\rho_o}\frac{d\rho}{dy}$. This same length a is useful for the inviscid case.

For steady withdrawal flows from a linearly stratified reservoir the following formulas are recommended for the thickness of the flowing layer (δ) in terms of the characteristic length a at distance x from the outlet:

a. Inviscid flow result (very close to the dam):

$$\frac{\delta}{a} = 2.7 \pm 0.2$$

$$\text{where } a = (q/\sqrt{g\varepsilon})^{1/2}$$

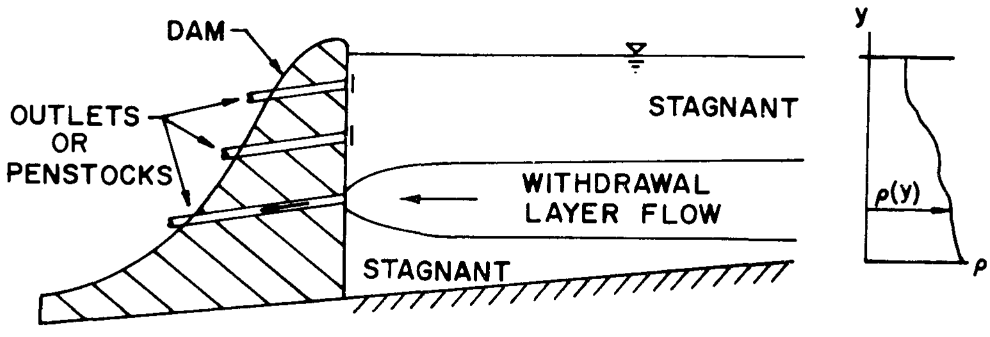


Figure 21. Selective withdrawal from a reservoir through one of several outlets at various levels in a dam. (After Brooks and Koh, 1969, C-1.)

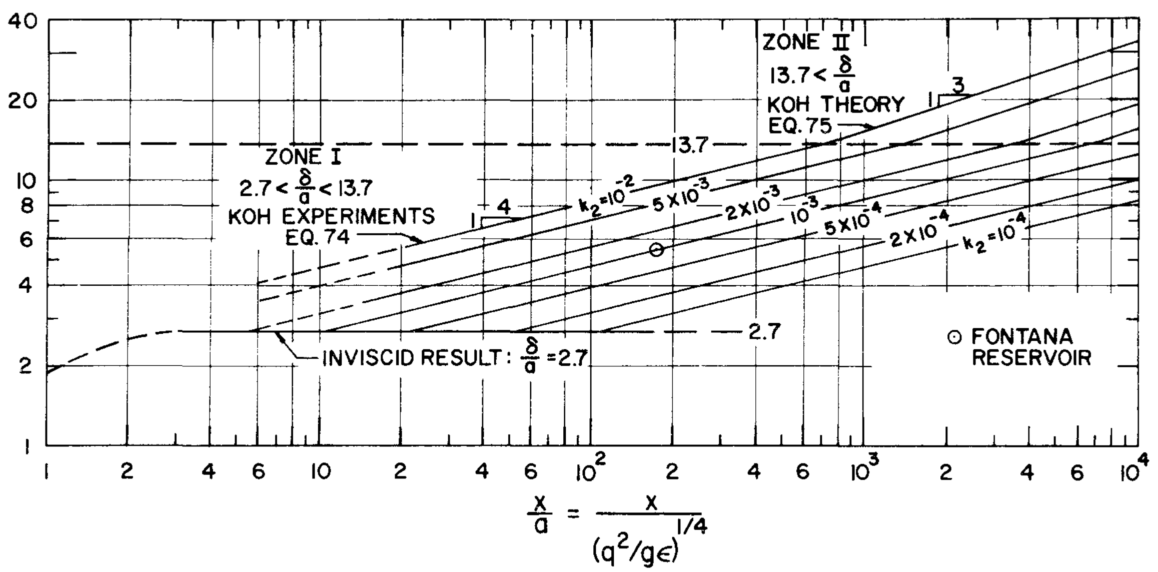


Figure 22. Summary of recommended formulas for selective withdrawal. (After Brooks and Koh, 1969, C-1.)

b. Turbulent flow, moderate distances (estimated on basis of Koh's experiments):

$$\frac{\delta}{a} = 8.4\left(k_2 \frac{x}{a}\right)^{1/4} \tag{74}$$

$$\text{for } 2.7 < \frac{\delta}{a} < 13.7$$

c. Turbulent flow, large distances (estimated on basis of Koh's theory):

$$\frac{\delta}{a} = 7.14 \left(k_2 \frac{x}{a}\right)^{1/3} \tag{75}$$

$$\text{for } \frac{\delta}{a} > 13.7$$

The relationship of these formulas is shown in Fig. 22.

The parameter $k_2 = E_m/q$ can only be determined from field experiments. It is anticipated that k_2 is of the order 10^{-3}.

Transients in the withdrawal currents caused by fluctuating power releases are predicted to be very persistent, taking many hours to die out. Transients may be thought of as very slow-moving internal waves. They will undoubtedly greatly complicate field measurements and management techniques in the future.

As a technique for water quality management, selective withdrawal has probably been somewhat overrated for continuously stratified flows. The temperature of water withdrawn at an outlet may be that of the water-column at that level, but, nonetheless, the discharge is a blend of water from the entire withdrawal layer thickness, δ. Typical values of δ are apt to range from 10's to 100's of feet. The withdrawal will be most selective when δ is small, or when the unit discharge q is relatively small and the stratification ($g\varepsilon$) is very strong.

Artificial Mixing in Stratified Reservoirs

A stratified water reservoir may be mixed by pumping water from one level to the other as shown in Fig. 23. At the intake there is essentially a selective withdrawal pattern while at the outlet a buoyant jet rises to its equilibrium level in the ambient stratification. As this jet rises it entrains bottom water and lifts it to the level where the plume spreads out laterally.

Ditmars (1970, C-2) developed a computer simulation for this process using short time steps. He assumed the selective withdrawal and buoyant-jet parts of the flow could be analyzed separately and that they were occurring in a quasi-static reservoir. In other words after a time step of pumping, the density profile would be adjusted to reflect the displacements resulting from the pumping. The constant density surfaces in the reservoir were assumed horizontal. The models for simulating the withdrawal and jet discharge were those developed during this project (primarily Brooks and Koh, 1969, C-1, and Ditmars, 1969, A-3).

The results of such a simulation are illustrated in Fig. 24 and compared with a laboratory experiment in a tank 9 m long, 45 cm deep and 61 cm wide. The levels of the intake and discharge pipes are shown at the right of the graph. The variable T is the characteristic time defined as $T = V/Q$, where V is the volume of tank between levels of intake and discharge and Q is the pumping rate. The real time elapsed from the start of pumping (t) is normalized by dividing by T. The agreement between theory and experiment is reasonably good. (The measured density profile at $t = 0$ is taken as the initial condition for the simulation, so perfect agreement is indicated at $t = 0$.) The effect of the jet is seen to be an ever-increasing uniformly mixed layer at the bottom. The rising plume generally stops rising and spreads out where the density curves suddenly change slope.

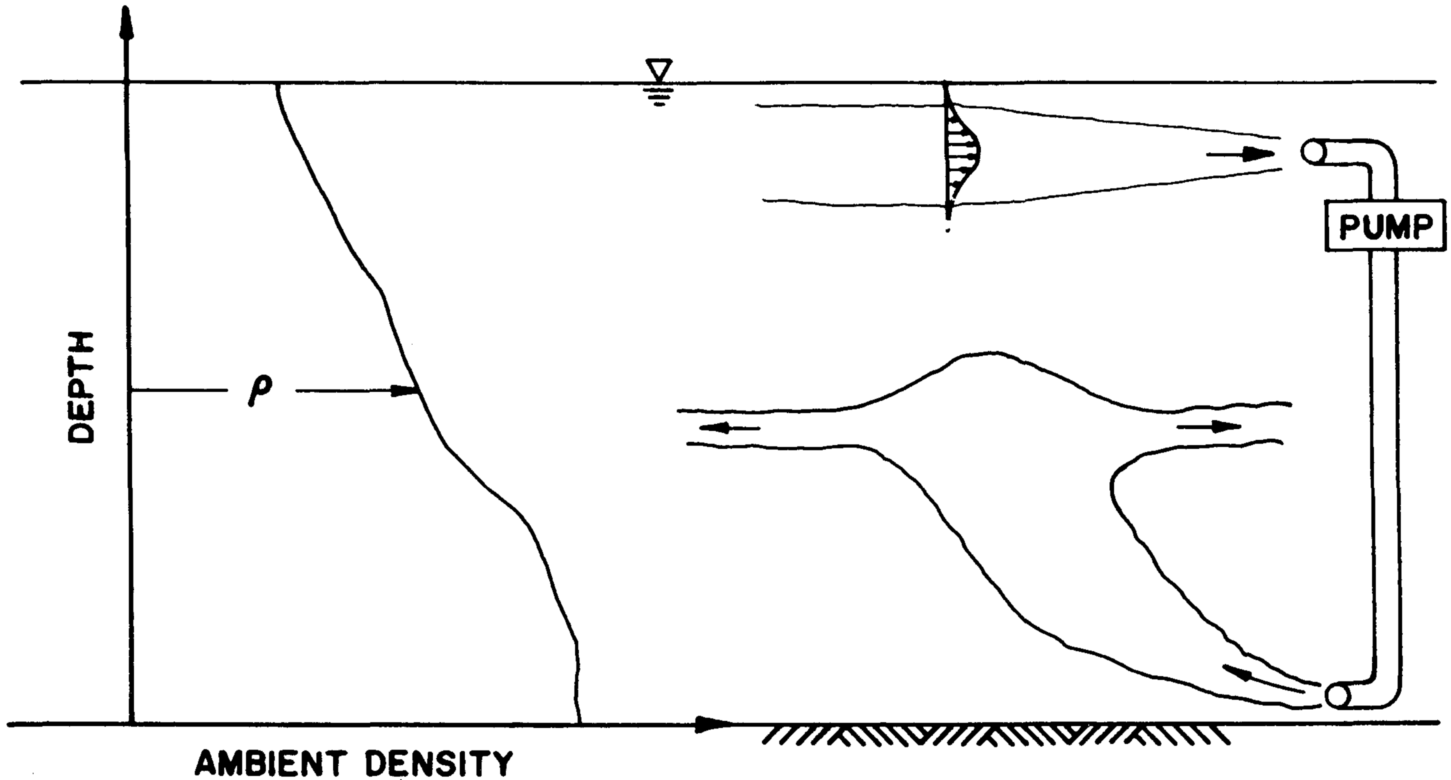


Figure 23. Schematic diagram of a pumping system for mixing a density-stratified reservoir. (After Ditmars, 1970, C-2.)

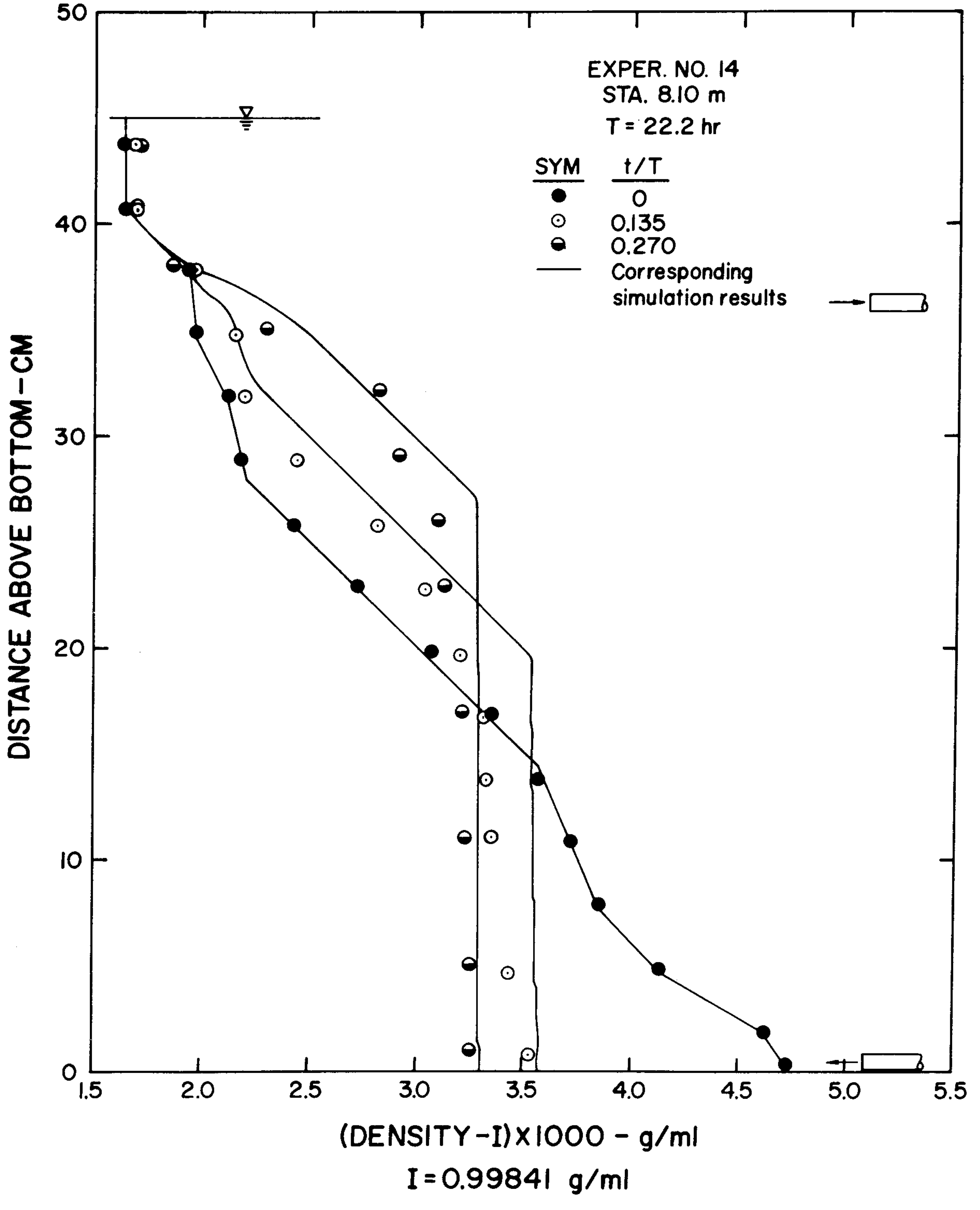


Figure 24. Measured and simulated density profiles for a typical experiment. (After Ditmars, 1970, C-2.)

The simulation analysis can be made for a reservoir of any geometric shape and any stable density profile at the start. However, to understand the sensitivity of the procedure to various parameters, it was found desirable to develop some generalized results for linear density profiles and reservoirs of constant surface area (vertical walls). The principal variables are as follows:

V = volume of water between levels of pump intake and discharge

d = depth, intake to discharge

$g\Delta\rho_o$ = difference in ambient weight density between outlet and inlet levels

= initial buoyant unit weight of jet discharge

ρ_o = reference density (at intake)

Q = pumping rate

D = discharge jet diameter

y = elevation above the discharge pipe

t = time since beginning of pumping

$g(\rho-\rho_o)$ = buoyant weight at any y and t

The density profile may now be expressed in dimensionless form as

$$\rho* = f(y*, t*, F, S, P) \tag{76}$$

wherein

$$\rho* = \frac{\rho-\rho_o}{\Delta\rho} \tag{77}$$

$$y* = y/d \tag{78}$$

$$t* = \frac{tQ}{V} \tag{79}$$

$$F = \frac{Q}{\frac{\pi}{4} D^2 \sqrt{g \frac{\Delta\rho}{\rho_o} D}} = \text{Froude number of the jet} \tag{80}$$

$$S = \frac{V}{d^3} \tag{81}$$

$$P = Q(g \frac{\Delta\rho}{\rho_o})^{-1/2} d^{-5/2} \tag{82}$$

The variables F, S, and P are fixed parameters for each flow.

A typical result of these non-dimensional simulations is shown in Fig. 25. From various simulation runs it was found that the results were not very sensitive to F, the jet Froude number. The mixing was slightly more effective for higher F values, but only at greatly increased energy input and lower efficiency. Thus most of the simulations to investigate the importance of P and S were made at a low Froude number, F = 3.

A convenient measure of the degree of destratification is the increase of potential energy associated with the redistribution of mass. In the stratified condition the center of mass is slightly below the center of volume, to which it is raised after complete mixing. The work required to do this is called "required potential energy increase." The successive steps of destratification may then be characterized by the fraction of this potential energy increase achieved at any time t*. Figure 26 gives the results to M as a function of t* and P for three values of S, and F = 3. The time required to reach a given degree of mixing (such as M = .90) is virtually independent of S, the shape parameter; for large S-values the reservoir has a very large area compared to the depth squared. The real time for pumping, t = t*V/Q, is directly proportional to the volume, but otherwise the shape has no direct effect because we assumed density surfaces to be flat (which cannot be exactly true for large reservoirs).

The sensitivity of M to P is also small. The parameter P indicates the local strength of the jet in relation to the density stratification: for small P the maximum height of rise of the buoyant jet (at t = 0+) is small compared to the total depth, and the converse for large P.

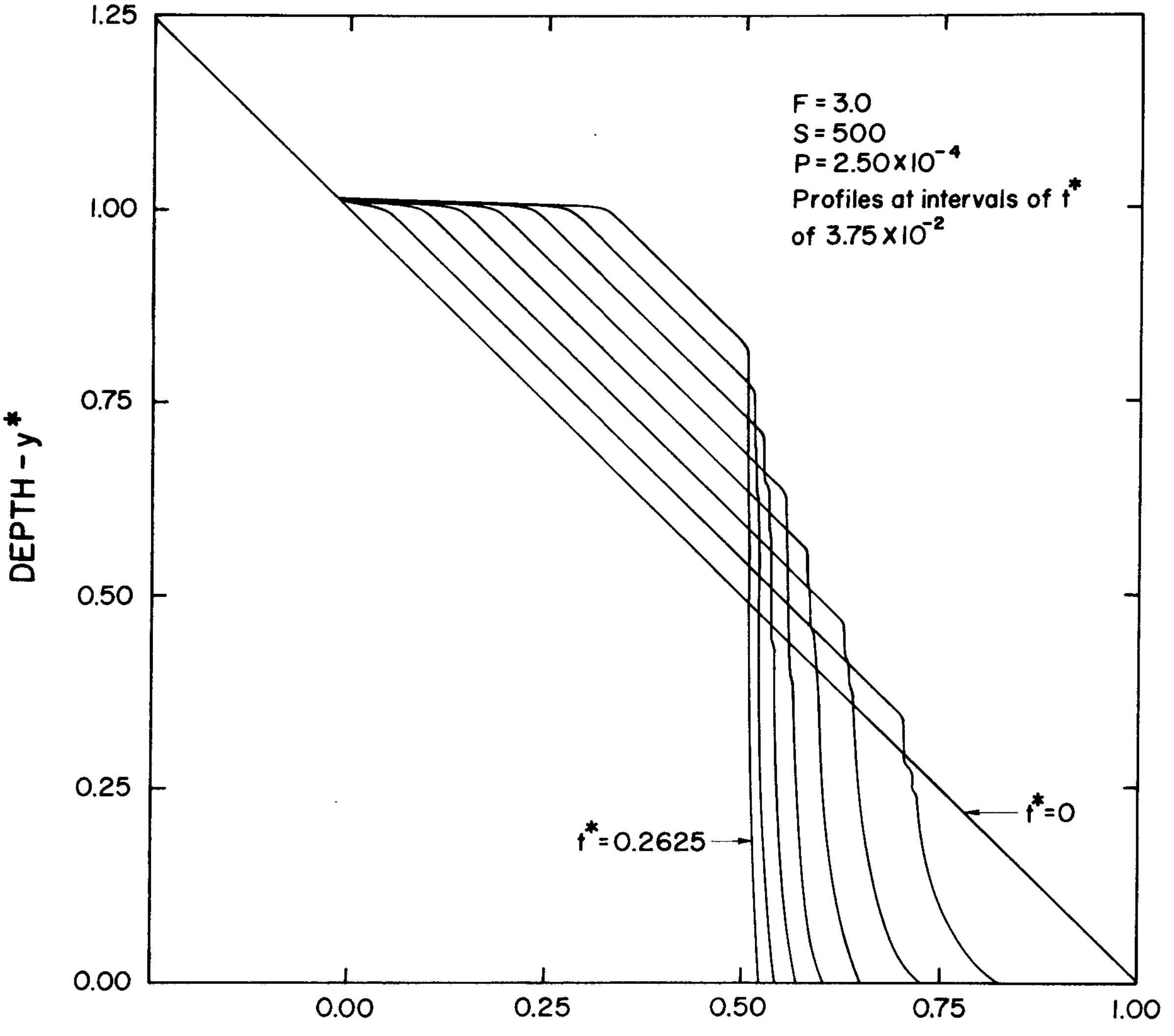


Figure 25. Typical non-dimensional density profiles by simulation of reservoir destratification by pumping (for S = 500, P = 2.5 x 10^{-4}). (After Ditmars, 1970, C-2.)

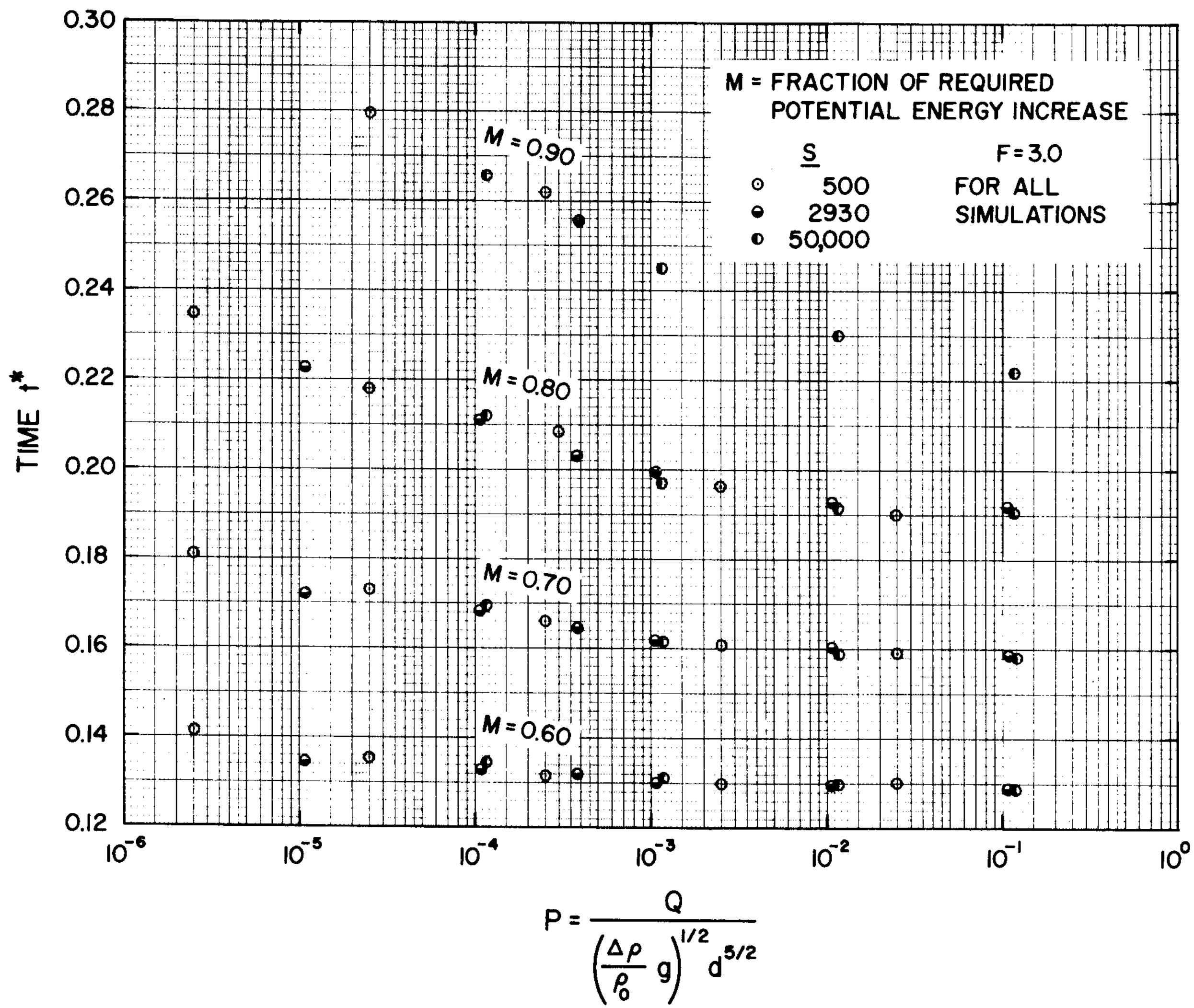


Figure 26. Summary of generalized simulation results for destratifying reservoirs by pumping: fraction of required potential energy increase (M) as a function of P and t*. (After Ditmars, 1970, C-2.)

In general, we may say that Fig. 26 implies that it takes $t^* \approx 0.21 \pm 0.02$ to achieve $M = .80$ (or close to complete mixing) regardless of the values of the parameters S, P, and F. The real time required is thus

$$t_1 = 0.21T = 0.21V/Q$$

Therefore, any pumping system has to be designed to pump at least 21% of the entire reservoir volume through the system in the time set to accomplish the mixing. The remaining water is moved by entrainment or displacement.

The mixing may be accomplished by one or several pumping systems, inasmuch as the result is insensitive to the parameter P. Even the provision of multiple-port diffusers would probably be of little use after the bottom water becomes fairly uniform. Furthermore jetting the water at high velocity is of little value. The lack of success of some previous field attempts (see Ditmars, 1969, C-2, Ch. 5) seems to be due to lack of adequate pumping capacity. However, for very large lakes the provision of very large pumping systems becomes very costly, as the required pumping rate may easily exceed the mean river flow.

This simulation does not include the effects of heat additions or losses during the mixing process. These factors can be added to the simulation as desired.

Another way to mix a stratified reservoir, or to reaerate it is to bubble air or oxygen. Cederwall and Ditmars (1970, A-5) made an analytical study of air bubble plumes, using the same integral techniques as for buoyant plumes treated previously. The mixture of water and bubbles was considered equivalent to a fluid of lighter density (for short time period). The results showed only fair agreement with previously published experimental results by others.

SECTION IX

TRANSVERSE MIXING IN RIVERS AND OTHER SHEAR FLOWS

In rivers and estuaries it is often important to know how rapidly pollutants will spread laterally across a flow. For example, if an outfall discharges near the middle of a river, how far downstream will it be before the pollutant is well mixed across the river (according to some set criterion)? And what is the pollutant concentration along the bank at various distances downstream? It is possible to get approximate answers to these questions by the classical diffusion equation, provided we can predict a good value of the diffusion coefficient for transverse mixing. The case of a pollutant of different density from the river requires special attention because of the temporary secondary circulations induced by the discharge.

This research project included several studies of transverse mixing, as reported in the Appendix, Section E. In the following sections, some important results are given.

Transverse Mixing -- No Density Difference

A major part of Okoye's work (1970, E-2) was concerned with measurement of the lateral diffusion coefficient in open channel flows in the 40-meter- and the 18-meter-long tilting flumes in the Keck Laboratory. The transverse mixing coefficient was determined from measurements of the distribution of a salt tracer downstream from a continuous source (small tube near middle of cross section). The salt solution was made neutrally buoyant by the addition of methanol. Fig. 27 shows the flow pattern schematically.

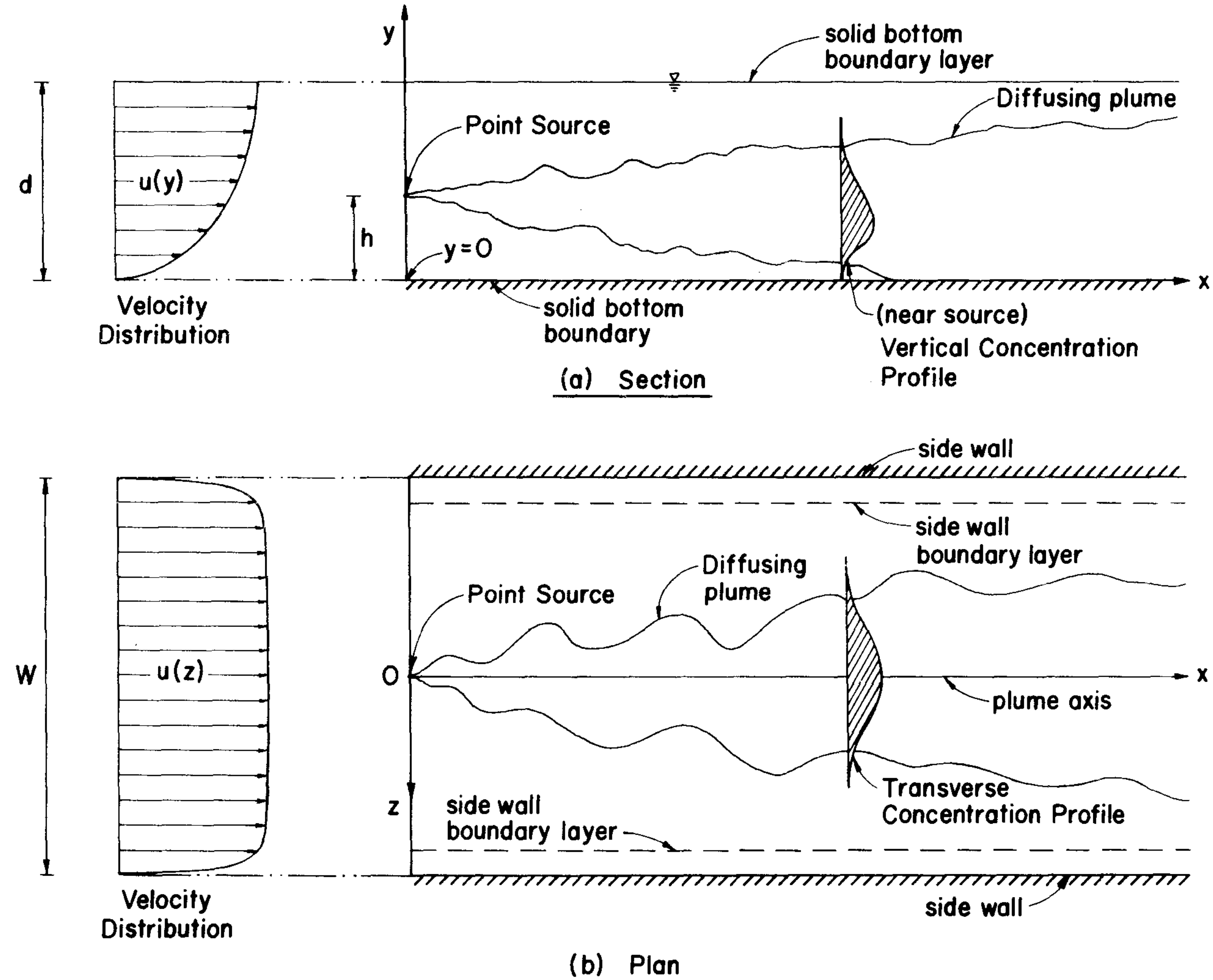


Figure 27. Definition sketch of plume geometry and coordinate axes. (After Okoye, 1970, E-2.)

The depth-averaged coefficient of transverse mixing $\bar{D}_z$ was defined and calculated by the relation:

$$\bar{D}_z = \frac{1}{2}\,\bar{u}\,\frac{d\overline{\sigma^2}}{dx} \tag{83}$$

where $\bar{u}$ = mean flow velocity in the flume cross section; σ^2 = variance of the concentration profile in transverse direction at any level in the flow; $\overline{\sigma^2}$ is the value of σ^2 averaged over the depth; and x is the distance downstream from the source. The theoretical basis for this relation in a flow with both vertical and horizontal mixing is given in detail by Okoye (1970, E-2). The coefficient thus defined includes both turbulent diffusion and transverse dispersion (Taylor type) associated with the secondary currents. The value of $\overline{\sigma^2}$ increased at a linear rate, with distance downstream, thereby making $\bar{D}_z$ independent of x.

The mixing coefficient is normalized by dividing by the product of u_{*b}, the shear velocity for the bottom, and d the depth:

$$\bar{\theta} = \frac{\bar{D}_z}{u_{*b}d} \tag{84}$$

Fig. 28 summarizes the results of the present study by Okoye and compares them with other laboratory and field measurements. It was found that the value of $\bar{\theta}$ is a function of the depth-width ratio, $d/w = \lambda$. The values of $\bar{\theta}$ ranged from 0.24 (at $\lambda = 0.015$) to 0.093 (at $\lambda = 0.20$) in the laboratory. The few values measured in the field by others are about twice as large, presumably because of stronger secondary currents caused by bends or channel irregularities.

For comparison it may be noted that the value of $\bar{D}_y/u_*d$, the normalized vertical mixing coefficient, is $\kappa/6$ (κ = von Karman constant) or 0.07. Thus in the flume the transverse mixing coefficient is of the order of 1.3 to 3 times larger than the vertical mixing coefficient.

The original report by Okoye also gives results for variation of D_z in the vertical; near-source spreading in the vertical direction; cross-sectional maps of tracer concentration; decay of maximum concentration; and position of point of maximum concentration.

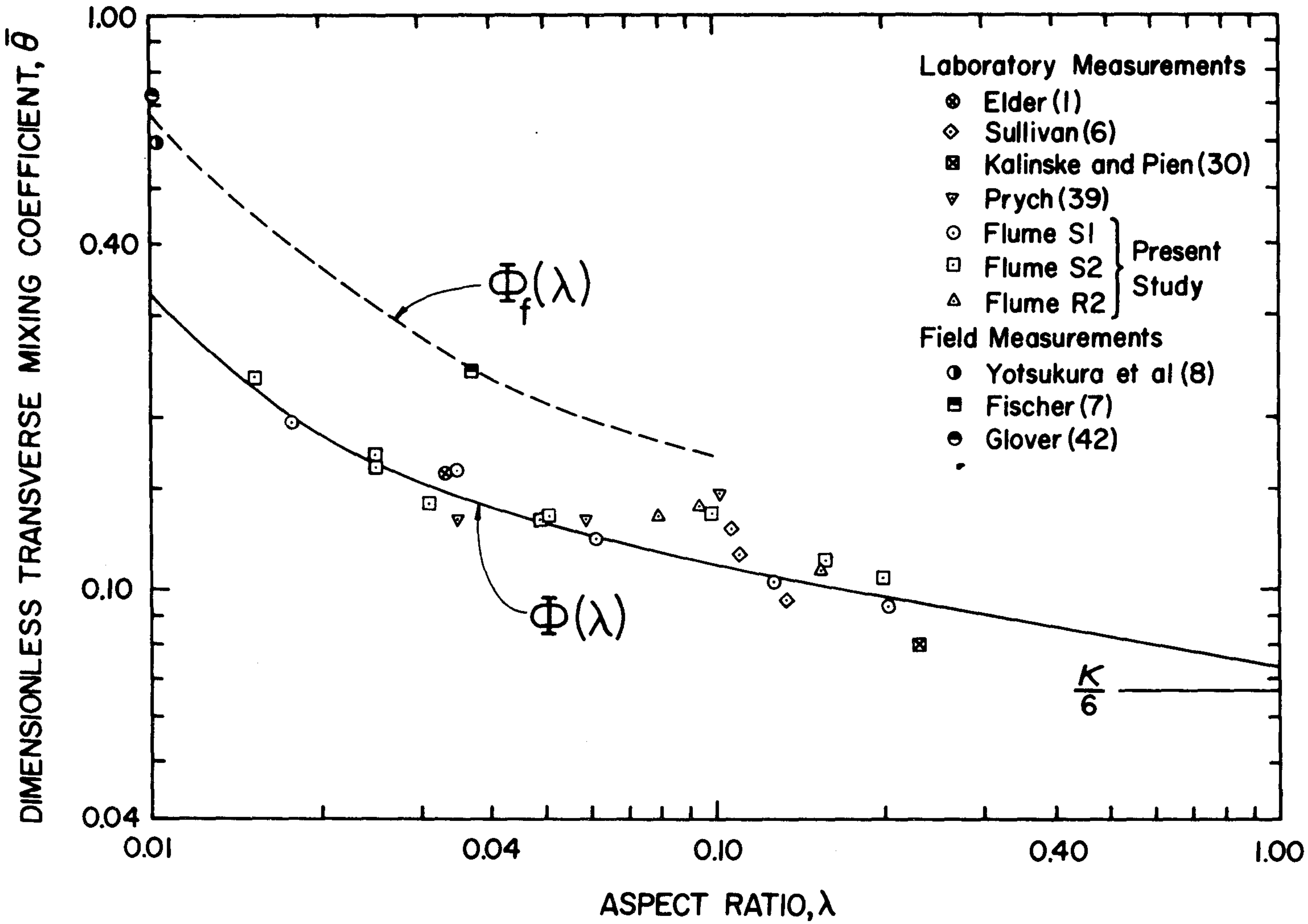


Figure 28. Variation of the depth-averaged dimensionless mixing coefficient $\overline{\theta}$ with the aspect ratio λ for all experiments performed in the present and past studies. (After Okoye, 1970, E-2.)

The second phase of Okoye's work dealt with phenomena associated with fluctuations in the spreading plume phenomenon. For a fluctuating plume, Fig. 29 illustrates the concept of regions of intermittency. Points within these regions are within the plume part of the time; the fraction of the time within the plume is called the intermittency factor, I_f. Within the plume there is an inner core for which $I_f = 1$, meaning that the region is always within the plume.

The growth of these zones is shown in Fig. 30 for a typical experiment; W_f is the extreme limit ($I_f = 0$) measured from the central axis; Δ is the edge of the continuous inner core ($I_f = 1$); and $\tilde{Z}$ is the mean position of the edge of the plume ($\bar{I}_f \approx 0.5$). For all the flume experiments (wide rectangular channels), these zone widths fit the following dimensionless relations:

$$\left(\frac{W_f}{d}\right)^2 = 3.6\, R_w \left(\frac{\chi}{d}\right) \frac{u_{*b}}{\bar{u}} \tag{85}$$

$$\frac{\tilde{Z}}{d} = 3.3\, R_z \left(\frac{\chi}{d}\right)^{2/3} \frac{u_{*b}}{\bar{u}} \tag{86}$$

$$\Delta = 2\tilde{Z} - W_f \tag{87}$$

where $R_w = (f_s/f_r)^{1/4}$

$R_z = (f_s/f_r)^{1/3}$

f_s = bed friction factor (smooth)

f_r = bed friction factor (observed)

χ = value of x corrected slightly for virtual origin of W_f

u_{*b} = shear velocity for the bottom

$\bar{u}$ = mean velocity

d = depth

Also investigated were the fluctuation frequencies for the plume as a whole, and the characteristics of concentration fluctuations with time at a point.

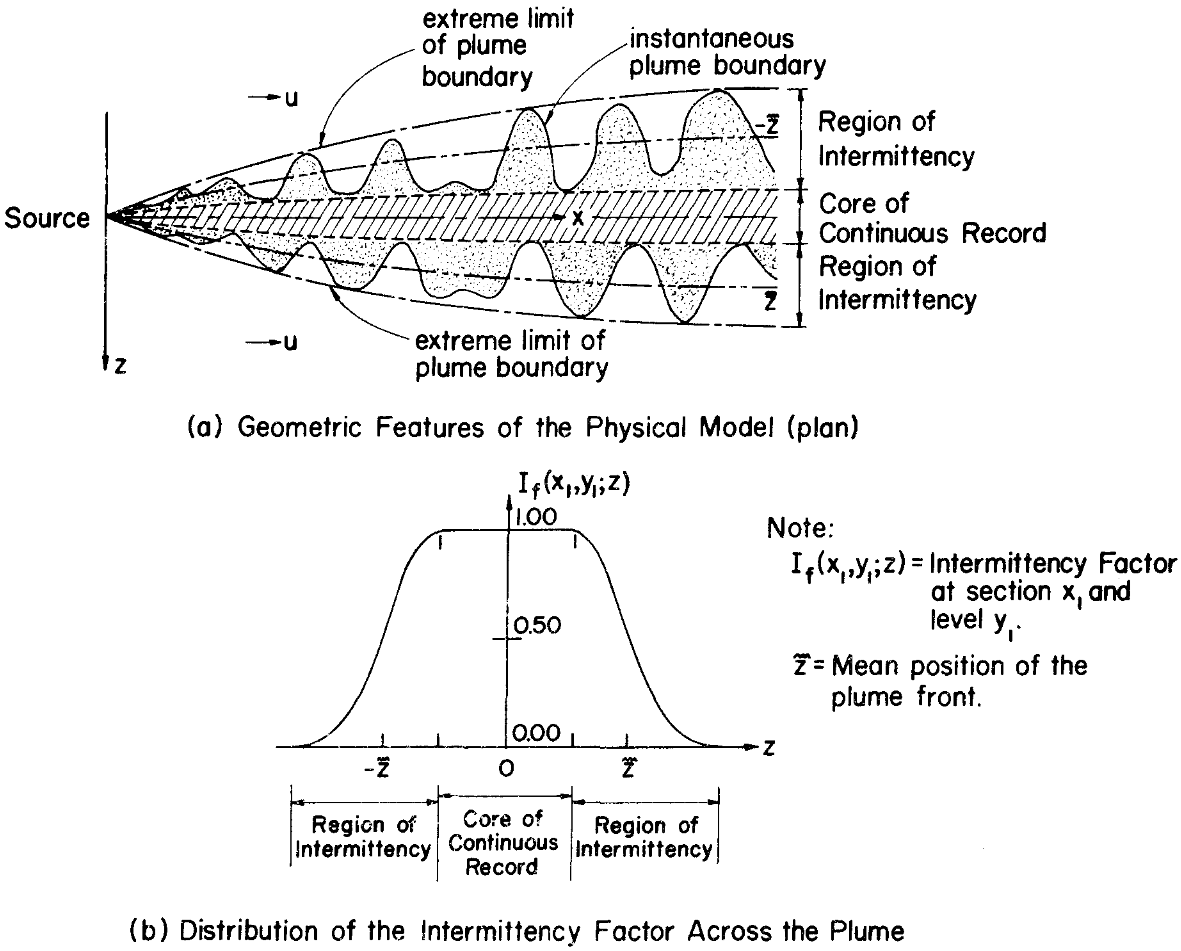


Figure 29. The intermittency factor model for cross-wise plume variation. (After Okoye, 1970, E-2.)

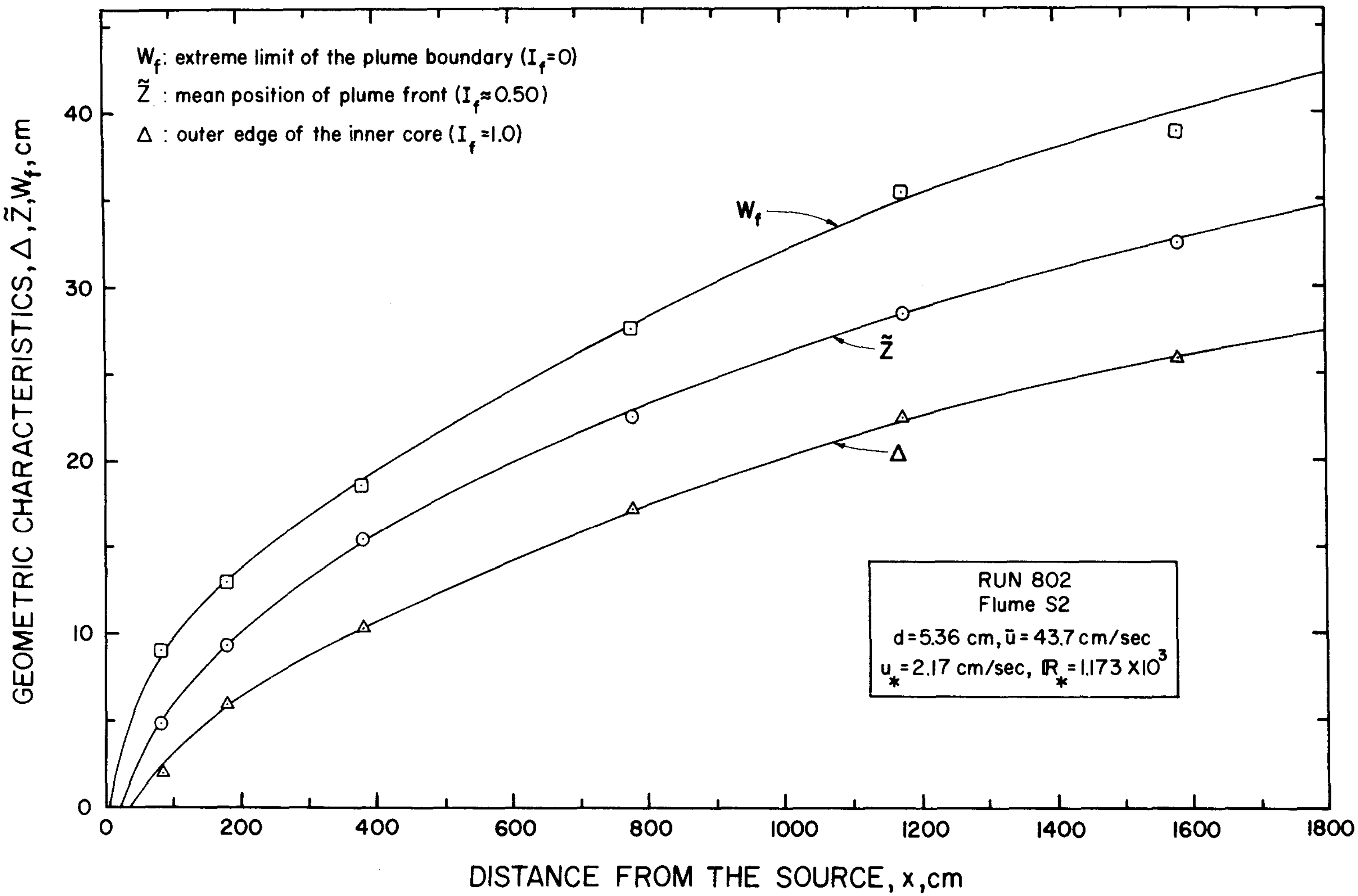


Figure 30. Growth of the geometric characteristics of the region of intermittency: RUN 802. (After Okoye, 1970, E-2.)

Transverse Mixing with Density Differences

If the pollutant flowing into a river or estuary is either heavier or lighter than its surroundings, it will induce a strong secondary flow pattern by sinking (or rising). This effect will accelerate the transverse spreading of the pollutant, but gradually vertical mixing will spread the pollutant uniformly over the depth and the driving force of the density difference approaches zero. The transverse mixing coefficient approaches that for a normal shear flow without density difference.

Prych (1970, E-1) presented the results of a comprehensive study on this subject. The basic flow pattern in shown in Fig. 31: within the 40-meter tilting flume (110 cm wide) a tracer stream was introduced through a slot of width b at approximately the same velocity as the stream flow.

Fig. 32(a) is an overhead photograph of the spreading of the tracer in a flume experiment when the density difference was zero ($\frac{\Delta\rho}{\rho} = 0$). Point measurements of concentration were made at different stations downstream as shown in Fig. 33. Measurements of the transverse mixing coefficient without density difference were made for all runs (by Eq. 83) to establish a basis for comparison with density runs; the results agreed well with Okoye's results (see Fig. 28).

For the case of a buoyant tracer stream ($\frac{\Delta\rho}{\rho} = -0.0158$), the spreading is much more rapid especially near the source as shown by Figs. 32(b) and 34. Note especially the evidence of the strong transverse surface current away from the source caused by the density difference.

For a quantitative description of the phenomenon, Prych computed the second moments of the transverse concentration distributions (averaged over the depth), and plotted these against distance downstream, x (see Fig. 35). The σ^2 for flow with hot water (Exp. 128) grew very rapidly at first, but then slowed to the same growth rate ($\frac{d\sigma^2}{dx}$) as the uniform density case (lowest curve, Exp. 116); other curves for heavy tracer fluid do not show such rapid spreading.

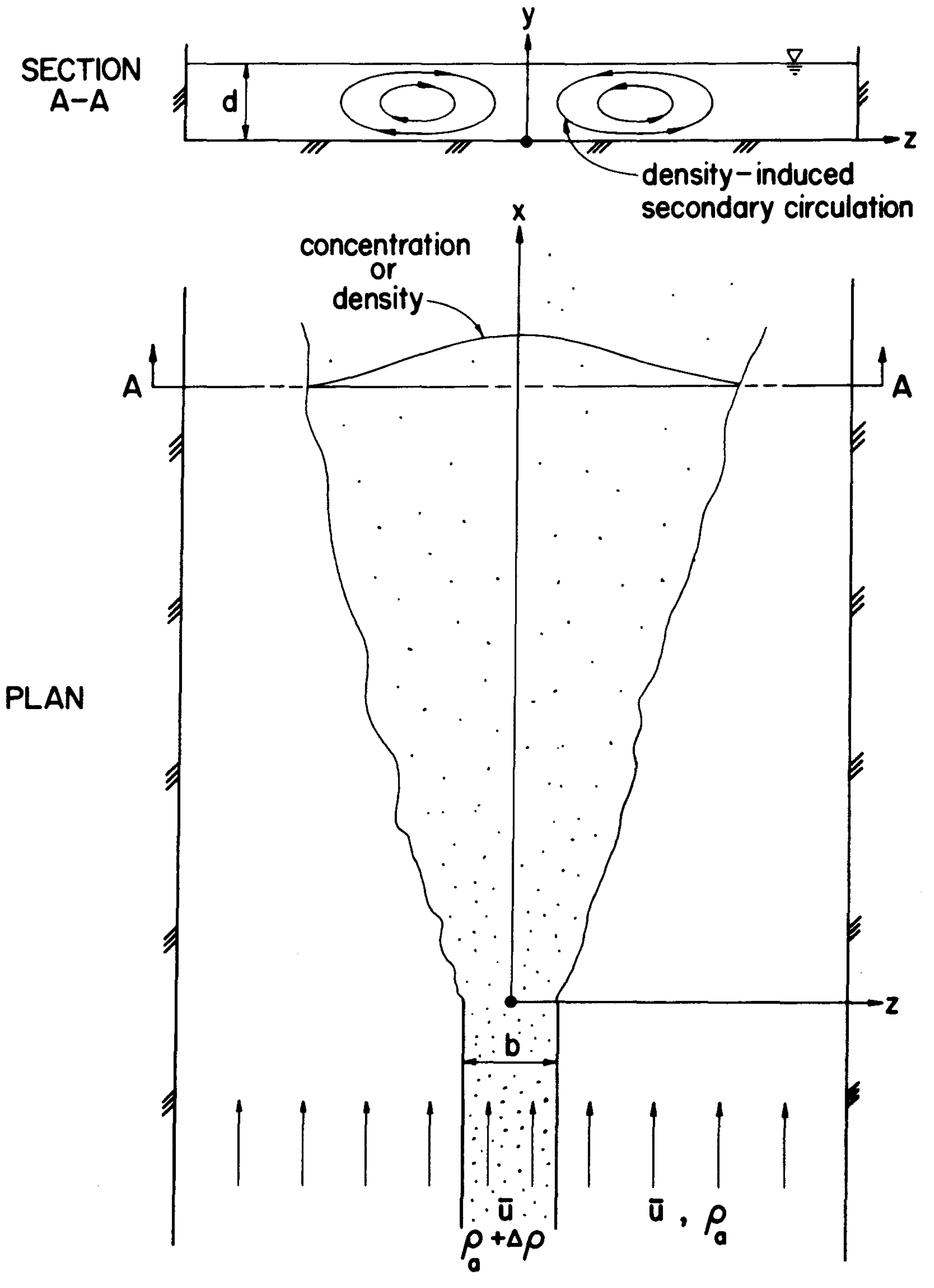
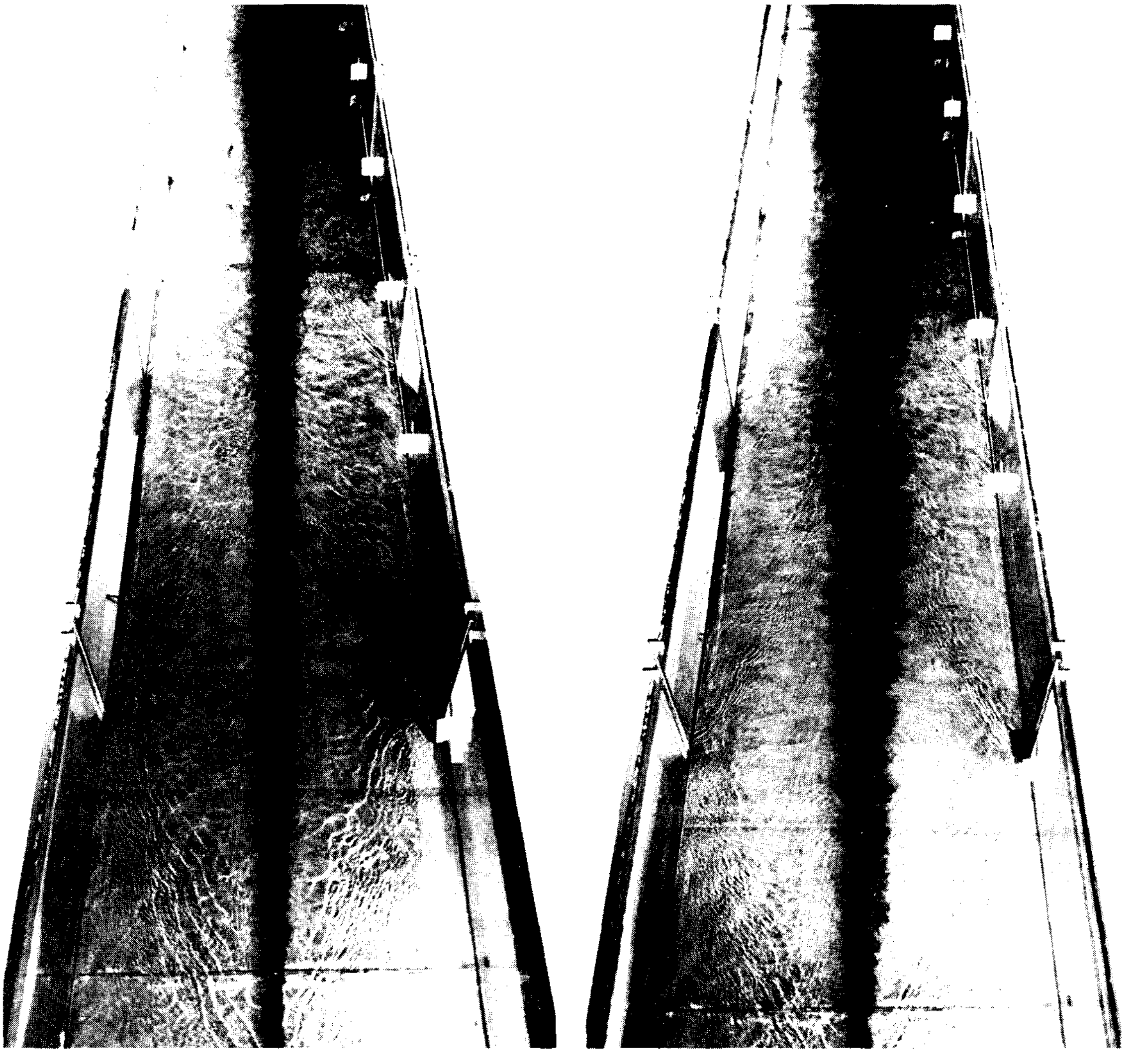


Figure 31. Definition sketch from transverse mixing experiments in laboratory channel. (After Brooks, 1970, G-1.)



(a) (b)

Figure 32. Overhead photograph of experiments in 110-cm wide flume with 1-cm wide source. (Depth = 6.55 cm, mean velocity = 45.2 cm/sec, shear velocity = 2.27 cm/sec.) (After Prych, 1970, E-1.)
(a) Exp. 116, $\Delta\rho/\rho$ = 0 (no density difference).
(b) Exp. 128, $\Delta\rho/\rho$ = -0.0158 (buoyant tracer).

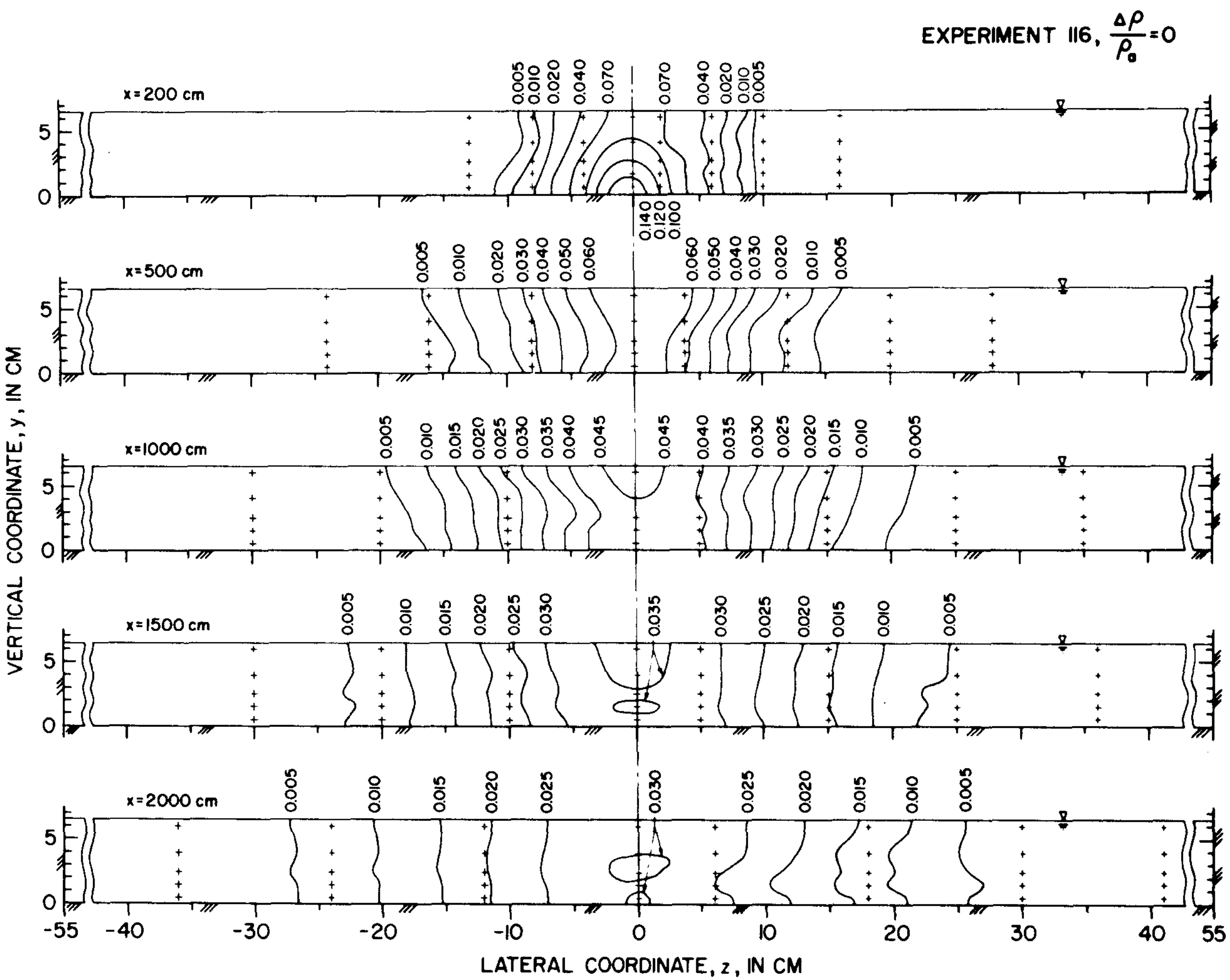


Figure 33. Contours of equal relative concentration in cross-sections downstream from a 1-cm wide source which discharged a fluid with a density the same as the ambient fluid and with a relative concentration of 1.0. The crosses designate sampling points. (After Prych, 1970, E-1.)

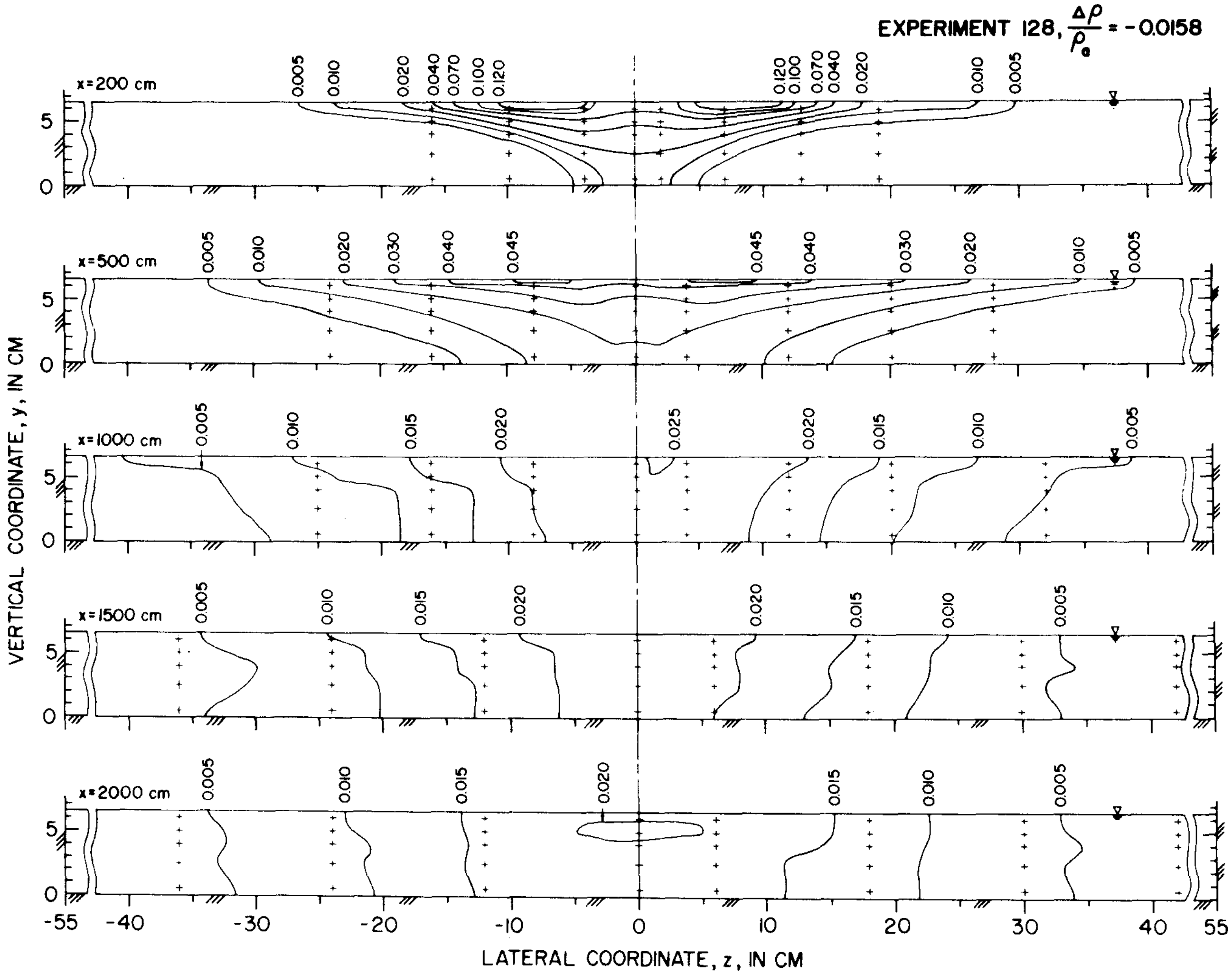


Figure 34. Contours of equal relative concentration in cross-sections downstream from a 1-cm wide source which discharged a fluid with a density 0.0158 gr/cm^3 less than the ambient fluid and with a relative concentration of 1.0. The crosses designate sampling points. (After Prych, 1970, E-1.)

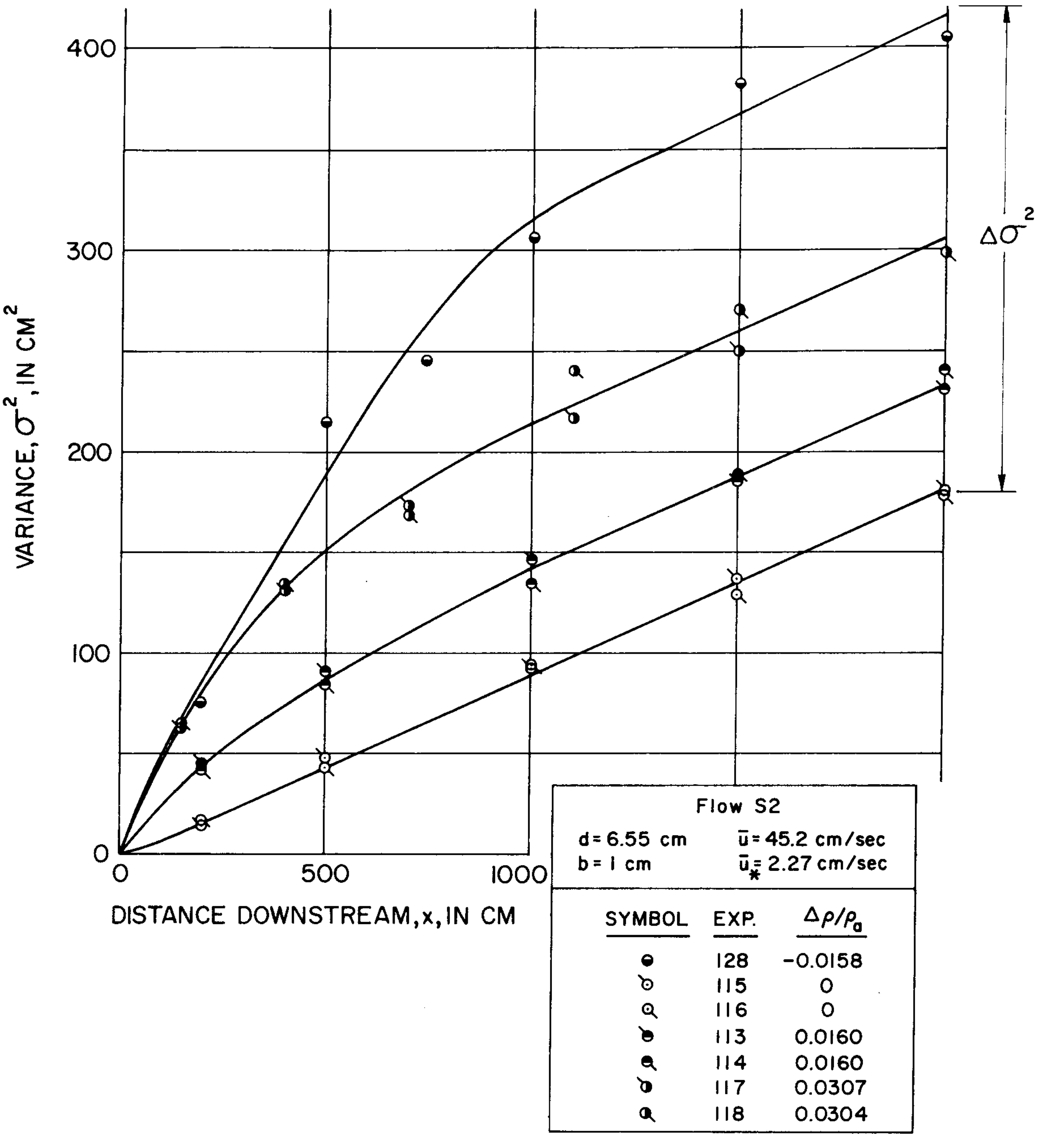


Figure 35. Variance - distance curves from flume experiments with 1-cm-wide source (smooth walls). (After Prych, 1970, E-1.)

The excess variance ($\Delta\sigma^2$) for density-stratified cases compared to uniform cases was summarized for many runs in a suitable dimensionless graph relating the important parameters.

The dimensionless variables used in Fig. 36 are:

$$\Delta V = \frac{\Delta(\sigma^2)}{d^2} = \text{dimensionless excess variance} \tag{88}$$

$\Delta\sigma^2$ = excess of transverse σ^2 for pollutant cloud over what it would be without density difference (depth-averaged)

$B = \frac{b}{d}$ = dimensionless source width

b = source width

d = depth

$$|M_b| = \frac{|\Delta\rho|}{\rho_a} gb/(\alpha u_*)^2 = \text{dimensionless source strength} \tag{89}$$

$\Delta\rho$ = density difference between injected and ambient fluids

ρ_a = ambient density

g = gravity

u_* = shear velocity

$\alpha = \overline{D}_z/u_* d$ = dimensionless transverse mixing coefficient

The curve in the upper left corner (dashed) is for buoyant tracers which spread faster than tracers which sink to the bottom and are retarded by bottom friction.

The straight line portions of the graph may be described by a single equation

$$\Delta V = \left(\frac{|M_b|}{M_b'}\right)^{3/2} \tag{90}$$

where M_b' is the intercept plotted in Fig. 37. For narrow sources (small B) the curve for buoyant injected fluid ($\Delta\rho/\rho_a < 0$) is different from the one for heavy injected fluid. Both curves are horizontal for narrow

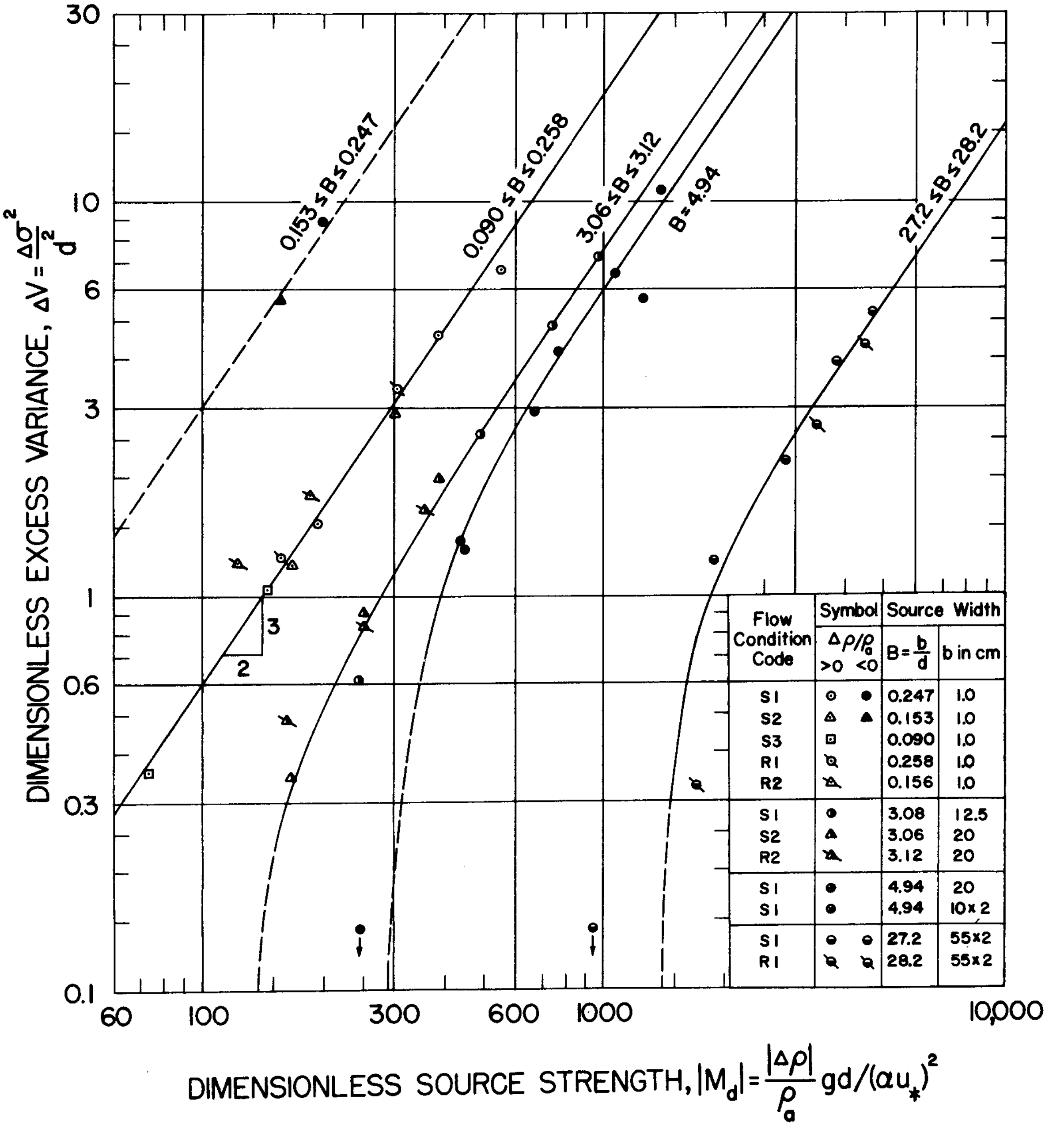


Figure 36. The dimensionless excess variance, ΔV, as a function of the dimensionless source strength, M_b, and the dimensionless source width, B. (After Prych, 1970, E-1.)

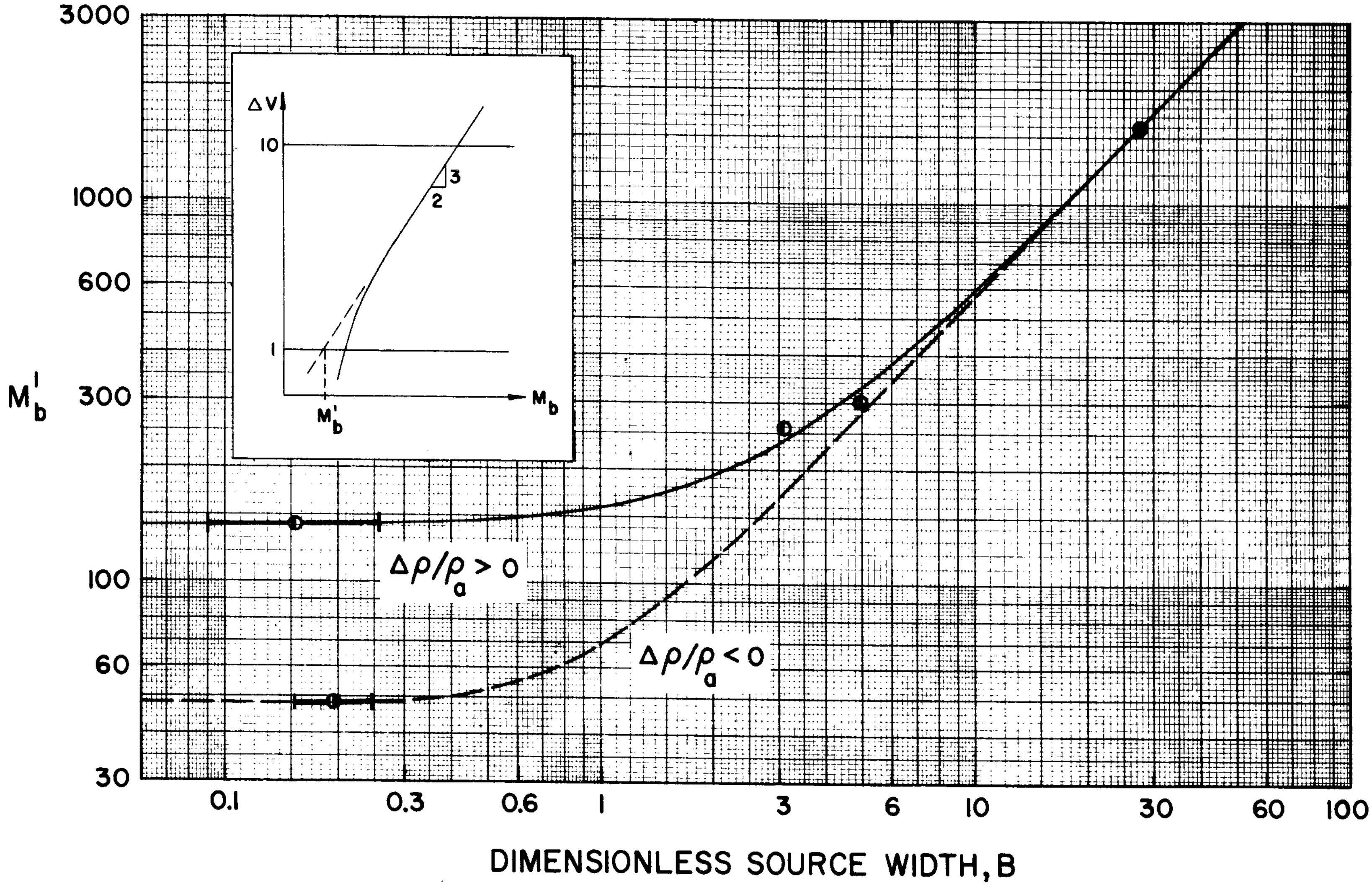


Figure 37. The intercept, $M_b^{\ 1}$, as a function of the dimensionless source width, B. (After Prych, 1970, E-1.)

sources ($B = b/D < 0.5$), because the lateral spreading depends only on the total strength of the buoyancy source without regard to the geometric ratio $B = b/d$. At the other limit, the $M_b' \propto B$, and M_b/M_b' becomes independent of b; in other words the source is so wide that its dynamics is controlled by the depth.

The excess variance is a one-time additional spreading of the pollutant cloud which occurs within an approximate distance of

$$X = \frac{x\alpha u_*}{\bar{u}d} < 1.5 \tag{91}$$

or

$$\frac{x}{d} < \frac{1.5\bar{u}}{\alpha u_*} \tag{92}$$

The dimensionless rate at which the excess ΔV grows is also given by Prych (1970, E-1).

Final Comment

The transverse spreading of a buoyant or heavy pollutant stream can be predicted from the results of this section. First the ordinary mixing coefficient $\bar{D}_z$ is obtained from Fig. 28 and Eq. 84; secondly the value of ΔV is determined from Fig. 37 and Eq. 90. Then

$$\sigma^2 = \frac{2xD_z}{\bar{u}} + d^2\Delta V + \sigma_o^2 \tag{93}$$

when $\overline{\sigma_o^2}$ is the initial value. Furthermore if $X < 1.5$, ΔV must be replaced by $r\Delta V$ where $r(<1)$ is a function of X given by Prych (1970, E-1). The excess spreading is relatively most important near the source.

Other research in mixing in channel flows undertaken in this project include Cederwall's work (1971, E-3) on lateral dispersion of floats. As the float size increased, the lateral diffusion coefficient decreased; for the largest floats (16 cm in diameter in the 110-cm-wide flume) the diffusion coefficient was 40% reduced from Okoye's values for solutes.

Coudert (1970, E-4) provided a very useful numerical program for solving the problem of diffusion in two dimensions (vertically and downstream) from a point source in two dimensions (i.e. line source across the stream in z-direction). With it he examined the near-source effects of non-uniform profiles of velocity and diffusion coefficient. For example, the locus of y-values for the peak concentration at each downstream station initially sinks below the level of the source. Examination of these details proved helpful in the interpretation of Okoye's results (1970, E-2).

SECTION X

ACKNOWLEDGMENTS

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The contributions of all the other research collaborators whose names appear as authors on the publication list in Section XII were essential to the success of this project. To them I express my special gratitude for their capable work which made the project possible.

Staff assistance in the Keck Laboratory is acknowledged in each of the individual publications as appropriate; however, the writer wishes to express special thanks to Mrs. Pat Rankin for typing this report and preparing it for publication.

SECTION XI

REFERENCES

The list below does not include those references which are included in the list of Publications, Reports, and Technical Memoranda of this project (see Section XII). Many more references were given in the literature review of each of the publications listed in Section XII. However, some additional references to work published on jet and plume mixing are listed below. By and large these items appeared during the interval between the preparation of research reports and the preparation of this final summary report.

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SECTION XII

PUBLICATIONS, REPORTS, AND TECHNICAL MEMORANDA
(W. M. Keck Laboratory of Hydraulics and Water Resources, California Institute of Technology)

Dispersion in Hydrologic and Coastal Environments
(EPA Grant No. 16070 DGY)

May 1967-December 1971

(Note: Abstracts for all items are given in the Appendix, by subject shown in right-hand column.)

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1969	6.	Fan, Loh-Nien, and Brooks, Norman H., "Numerical Solutions of Turbulent Buoyant Jet Problems," W. M. Keck Laboratory of Hydraulics and Water Resources, Report No. KH-R-18, January 1969, 94 pp.	A-2

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7. Ditmars, John D., "Computer Program for Round Buoyant Jets into Stratified Ambient Environments," W. M. Keck Lab. of Hydraulics and Water Resources, Tech. Memo No. 69-1, March 1969, 27 pp. A-3

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1971 22. Cederwall, Klas, Discussion of "Horizontal Surface Discharge of Warm Water Jets," J. of the Power Division, ASCE, Vol. 97, No. PO1, Jan. 1971, pp. 229-234. A-10

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25. Sullivan, Paul J., "Some Data on the Distance-Neighbour Function for Relative Diffusion," Journal of Fluid Mechanics, Vol. 47, Part 3, June 14, 1971, pp. 601-607. D-2

26. Sotil, C. A., "Computer Program for Slot Buoyant Jets into Stratified Ambient Environments," W. M. Keck Laboratory of Hyeraulics and Water Resources, Tech. Memo 71-2, June 1971, 35 pp. A-11

27. Rumer, Ralph R., Jr., "Internal Seiches and Interfacial Mixing in Stratified Lakes," W. M. Keck Laboratory of Hydraulics and Water Resources, Tech. Memo 71-3, July 1971, 39 pp. D-4

28. Cederwall, Klas, "Float Diffusion Study," Water Research, Vol. 5, pp. 889-907, Nov. 1971. (Also W. M. Keck Laboratory of Hydraulics and Water Resources, Tech. Memo 71-1, April 1971.) E-3

29. Sullivan, Paul J., "Longitudinal Dispersion within a Two-Dimensional Turbulent Shear Flow," Journal of Fluid Mechanics, Vol. 49, Part 3, Oct. 15, 1971, pp. 551-576. E-6

1972 30. Sullivan, Paul J., "The Penetration of a Density Interface by Heavy Vortex Rings," Water, Air, and Soil Pollution, 1, 3, pp. 322-336, July 1972. A-8

1973 31. List, E. J., and Imberger, Jorg, "Turbulent Entrainment in Buoyant Jets and Plumes," J. of Hydraulics Div., ASCE, 1973 (in press). A-9

32. Rumer, Ralph R., Jr., "Interfacial Wave Breaking in Stratified Liquids," J. of Hydraulics Div., ASCE, Vol. 99, No. HY3, March, 1973, pp. 509-524. D-5

SECTION XIII

APPENDIX

ANNOTATED LIST OF PUBLICATIONS, REPORTS AND TECHNICAL MEMORANDA

The following documents describe in greater detail the research supported entirely or partially by this grant (16070DGY) during the period May 1, 1967 to December 31, 1971. In some instances, part of the research was done under previous grants of the National Institutes of Health, USPHS, with final publication during this period. Individual copies of most of these documents may be obtained on request to the Secretary, W. M. Keck Laboratory of Hydraulics and Water Resources, California Institute of Technology, Pasadena, California, 91109.

The documents are arranged by subject, with an abstract for each. The writer acknowledges the individual authors for most of the following abstracts, although some editorial revisions and additions have been made by this writer.

The major subject headings are as follows:

A. JET AND PLUME MIXING

A-1. FAN, LOH-NIEN, "Turbulent Buoyant Jets into Stratified or Flowing Ambient Fluids," W. M. Keck Laboratory of Hydraulics and Water Resources, Report KH-R-15, Caltech, June, 1967, 196 pp.

Theoretical and experimental studies were made on two classes of buoyant jet problems, namely:

1) an inclined, round buoyant jet in a stagnant environment with linear density-stratification;

2) a round buoyant jet in a uniform cross stream of homogeneous density.

Using the integral technique of analysis, assuming similarity, predictions were made for jet trajectory, widths, and dilution ratios, in a density-stratified or flowing environment. Such information is of great importance in the design of disposal systems for sewage effluent into the ocean or waste gases into the atmosphere.

This study of a buoyant jet in a stagnant environment extended the Morton type of analysis to cover the effect of the initial angle of discharge. Numerical solutions have been presented for a range of initial conditions. Laboratory experiments were conducted for photographic observations of the trajectories of dyed jets. In general the observed jet forms agreed well with the calculated trajectories and nominal half widths when the value of the entrainment coefficient was taken to be $\alpha = 0.082$, as previously suggested by Morton.

The problem of a buoyant jet in a uniform cross stream was analyzed by assuming an entrainment mechanism based upon the vector difference between the characteristic jet velocity and the ambient velocity. The effect of the unbalanced pressure field on the sides of the jet flow was approximated by a gross drag term. Laboratory flume experiments were performed with sinking jets, which are directly analogous to buoyant jets. Salt solutions were injected into fresh water at the free surface of the flow in a large flume. The jet trajectories, dilution ratios and jet half widths were determined by conductivity measurements. The entrainment coefficient, α, and drag coefficient, C_d, were found from the observed jet trajectories and dilution ratios. In the ten cases studied where jet Froude number ranged from 10 to 80 and velocity ratio (jet: current) k from 4 to 16, α varied from 0.4 to 0.5 and C_d from 1.7 to 0.1. The jet mixing motion for distances within 250D (D = initial diameter) was found to be dominated by the self-generated turbulence, rather than the free-stream turbulence. Similarity of concentration profiles has also been discussed.

A-2. FAN, LOH-NIEN, and BROOKS, NORMAN H., "Numerical Solutions of Turbulent Buoyant Jet Problems," W. M. Keck Laboratory of Hydraulics and Water Resources, Report KH-R-18, Caltech, January 1969, 94 pp.

Theoretical solutions were obtained on four classes of turbulent buoyant jet problems, namely:

1) an inclined, round buoyant jet in a stagnant, uniform ambient fluid;

2) an inclined, round buoyant jet in a stagnant ambient fluid with linear density-stratification;

3) an inclined, slot buoyant jet in a stagnant, uniform ambient fluid;

4) an inclined, slot buoyant jet in a stagnant ambient fluid with linear density-stratification.

This report is a summary of the numerical solutions on buoyant jets in stagnant environments carried out in connection with previous investigations at Caltech by Fan, Brooks, and Koh. Using the integral type of analysis assuming similarity, predictions can be made for jet trajectory, widths, and dilution ratios, in a uniform or density-stratified environment without ambient currents. Numerical solutions have been presented in dimensionless form for a wide range of initial conditions, including the effect of the initial angle of discharge. Since the integral analysis is only for the zone of established flow, adjustments are given for the effects of the zone of flow establishment for finite jets (i.e. the initial development of self-similar profiles).

Problems with non-linear density profiles are not readily treated in generalized non-dimensional form. Rather it is more feasible to make case-by-case calculations using dimensional variables. A program for such calculations for a round jet is available in a technical memorandum by Ditmars.

These solutions are useful in the design of submerged jet diffusers for disposal of sewage effluent into the ocean or cooling water into a lake.

A-3. DITMARS, JOHN D., "Computer Program for Round Buoyant Jets into Stratified Ambient Environments," W. M. Keck Lab. of Hydraulics and Water Resources, Tech. Memo. No. 69-1, Caltech, March 1969, 27 pp.

The gross behavior of an inclined round turbulent buoyant jet in a stratified ambient environment is determined by quadrature of the governing differential equations. The FORTRAN IV (level G) language is used for the program, which has been run on an IBM 360/75 digital computer. The essential input to the problem includes the location of the jet; the initial values of jet velocity, jet diameter, and angle of inclination; the density of the discharged fluid; and the density profile of the ambient environment. The density profile may have any gravitationally stable shape. It is provided as input to the program by supplying the density at arbitrarily selected points (which best fit the actual profile) and calculated as linear between these points. The output consists of the jet trajectory in rectangular coordinates; the nominal width; the centerline velocity, density difference, and dilution ratio. All of these parameters are printed out at uniform intervals along the jet trajectory. Calculations are stopped at the maximum height of rise or greatest depth of sinking of the jet or at any predetermined vertical or horizontal coordinate.

The program is applicable to several limiting cases of round turbulent jets. Horizontal, vertical, and inclined jets ($-90° \leq \theta_o \leq 90°$) with initially positive or negative buoyancies can be handled in stratified or uniform ambient environments. The jet may be placed at any elevation in the ambient environment. Simple momentum jets can be handled, although simple plumes are not amenable to this technique.

For cases of linear density profiles in the environment a variety of numerical computer solutions have been presented in generalized non-dimensional coordinates by Fan and Brooks. However, for arbitrary non-linear density profiles, generalized non-dimensional solutions are not feasible; it is for this reason that the computer program is being made available for case-by-case solutions.

A-4. FAN, LOH-NIEN, and BROOKS, NORMAN. H., Discussion of "Physical Interpretation of Jet Dilution Parameters," by James J. Sharp, Jour. of Sanitary Eng. Div., ASCE, Vol. 94, No. SA6, Dec. 1968, pp. 1295-1299.

This discussion explains the effect of the geometry of buoyant jet trajectories on the shape of the curves in the usual dilution graph (dilution contours plotted as function of Froude number, F, and normalized vertical coordinate, y/D). Dilution graphs are given for horizontal and vertical buoyant jets in uniform environment, according to the Fan and Brooks analysis. For low Froude numbers, both graphs approach the buoyant plume solution, whereas for high Froude numbers (high jet efflux velocities) the results are markedly different because of the much longer trajectories developed for the horizontal jets than for the vertical jets in rising a prescribed distance.

A-5. CEDERWALL, KLAS, and DITMARS, JOHN D., "Analysis of Air-Bubble Plumes," W. M. Keck Laboratory of Hydraulics and Water Resources, Report No. KH-R-24, Caltech, Sept. 1970, 51 pp.

The air-bubble plume induced by the steady release of air into water has been analyzed with an integral technique based on the equations for conservation of mass, momentum and buoyancy. This approach has been widely used to study the behavior of submerged turbulent jets and plumes, as for example by Fan and Brooks (see above). The case of air-bubble induced flow, however, includes additional features. In this study the compressibility of the air and the differential velocity between the rising air bubbles and the water are introduced as basic properties of the air-bubble plume in addition to a fundamental coefficient of entrainment and a turbulent Schmidt number characterizing the lateral spreading of the air bubbles.

Theoretical solutions for two- and three-dimensional air-bubble systems in homogeneous, stagnant water are presented in both dimensional and normalized form. Comparison with published data indicates fairly good agreement between theory and experiment. The further complication of a stratified environment is briefly discussed since this case is of great practical interest.

The paper is to be considered as a progress report, as future experimental verification of various hypotheses is needed.

A-6. CEDERWALL, KLAS, "A Buoyant Slot Jet into Stagnant or Flowing Environments," W. M. Keck Laboratory of Hydraulics and Water Resources, Report KH-R-25, Caltech, March 1971, 86 pp.

The diffusion following the release of a buoyant slot jet into a confined, uniformly flowing environment has been studied. A dimensional analysis reveals the complexity of the problem; there are in general four governing dimensionless numbers. For small mass flow from the source (the source then being characterized by initial flux of momentum, m, and buoyancy, b, only) the governing flow parameters reduce to:

1. Source Froude number $F = u_a^3/b$

2. Momentum flux ratio $= u_a^2 H/m$

3. Angle of discharge, θ

wherein u_a is the ambient velocity and H is the depth of the ambient flow.

Experiments were conducted first with a horizontal, buoyant slot jet into stagnant, ambient fluid. Observed trajectories and centerline dilutions were in good agreement with existing theories. Next two sets of experiments were performed with a vertical and a horizontal buoyant slot jet issuing into a uniformly flowing stream.

A two-layer flow analysis provided the rationale for a classification of flow regimes. It was found that the jet effluent cloud would penetrate upstream from the jet outlet whenever the source Froude number was less than about unity. For high source Froude numbers the jet is swept downstream, either as a wall jet (in spite of its buoyancy) or in an unstable highly turbulent pattern.

The results are applicable to the discharge of cooling water (or other buoyant waste water) from multi-port line diffusers into rivers, homogeneous estuaries, or shallow reservoirs wherein the jet diffusion is strongly influenced by the limited depth of flow.

A-7. LIST, E. J., "Laminar Momentum Jets in a Stratified Fluid," J. of Fluid Mechanics, Vol. 45, February 15, 1971, pp. 561-574.

Solutions are presented for creeping flows induced by two- and three-dimensional horizontal and vertical momentum jets in a linearly-stratified unbounded diffusive viscous fluid. These linear problems are solved by replacing the momentum jet by a body force singularity represented by delta functions and solving the partial differential equations of motion by use of multi-dimensional Fourier transforms. The integral representations for the physical variables are evaluated by a combination of residue theory and numerical integration.

The solutions for vertical jets show the jet to be trapped within a layer of finite thickness and systems of rotors to be induced. The horizontal two-dimensional jet solution shows return flows above and below the jet and a pair of rotors. The three-dimensional horizontal jet has no return flow at finite distance and the diffusive contribution is found to be almost negligible in most situations, the primary character of the horizontal flows being given by the non-diffusive solution. Stokes's paradox is found to be non-existent in a density-stratified fluid.

The results obtained for laminar momentum jets in a stratified fluid will be helpful in understanding turbulent jets in stratified fluid. This work differs from that previously discussed in this section in that: (a) there is no assumption of self-similarity of velocity profiles, and hence the dynamics of jet collapse and induced currents can be studied; and (b) the buoyancy or mass fluxes at the source are negligible, and the flow is driven solely by the momentum input.

A-8. SULLIVAN, PAUL J., "The Penetration of a Density Interface by Heavy Vortex Rings," Water, Air, and Soil Pollution, 1, 3, pp. 322-336, July 1972.

This paper describes experiments in which small volumes of heavy fluid were released in the uppermost of two uniform layers of fluid and the degree of penetration into the lower layer determined. When the injected fluid had no initial momentum, less than ten percent of the released fluid continued into the lower layer if $g \frac{Z'^3P}{F}$ was greater than 29; more than ninety percent continued into the lower layer if $g \frac{Z'^3P}{F}$ was less than 1.5. (Z' is the distance from release to the interface, $P = \frac{\rho_2 - \rho_1}{\rho_1}$ where ρ_1 and ρ_2 are the densities of the upper and the lower fluid and $F = g \frac{(\rho - \rho_1)}{\rho_1} V$ where V is the volume and ρ is the density of the injected fluid.)

When initial momentum was given to the heavy fluid, a vortex ring then formed, penetrated into the lower layer, broke up, and more than ninety percent of this fluid remained in the lower layer if $g \frac{Z'^3 P}{F}$ was less than 0.30 $(\frac{K^2}{F})^3$. The vortex ring was found to remain intact during its travel in the lower layer if $g \frac{Z'^3 P}{F}$ was less than 0.17 $(\frac{K^2}{F})^3$. (K is the initial circulation of the injected fluid.)

The results may tentatively be applied to the sudden dumping of sludge in the ocean, when the ocean density structure can be adequately approximated by two homogeneous layers of slightly different density. There is uncertainty, however, regarding the effect of Reynolds number on the critical dimensionless numbers cited above.

A-9. CEDERWALL, KLAS, "Dispersion Phenomena in Coastal Environments," J. of Boston Soc. of Civil Eng., Vol. 57, No. 1, Jan. 1970, pp. 34-70.

This paper, presented as the 1970 Freeman Memorial Lecture before the Boston Society of Civil Engineers, reviews a wide range of topics relating to dispersion of both sewage and thermal effluents in coastal waters with special emphasis on Swedish practice.

There are several alternative methods for the prediction of the dispersion pattern in the receiving water area. Four main approaches may be distinguished.

a) A purely theoretical analysis supported by general experience on the diffusive properties, circulation and exchange conditions of the water area in question.

b) The same as under a), but with supplementary field surveys to establish characteristic prototype behavior.

c) Tracer technique for direct in-situ simulation of transport and mixing of the waste effluent.

d) Scale tests by means of a hydraulic model.

For thermal discharges, the author describes two phenomena, diffusion of a submerged jet and surface spreading due to buoyancy. If a surface discharge jet is dominated by its momentum, it may be considered as approximately equivalent to half of a complete submerged jet. On the other hand a low-speed channel discharge, dominated by buoyancy rather than momentum, will spread laterally as a wave front. Other cases are intermediate, because they involve both turbulent entrainment and gravitational spreading. The effects of limited depth are also discussed, with the limiting situation being similar to a wall jet.

For sewage discharges into confined water bodies the initial mixing and diffusion may not be as important as the overall flushing of the water body, such as fjords and archipelagos. Flushing may occur naturally by density-induced circulation caused by fresh water runoff, by wind-driven currents, by tidal currents, or a combination. In each case the density structure and the vertical mixing are important. Because of the complexity of the phenomena, there is need for considerable input of field data into any theoretical models. Therefore, it is often more effective to make tracer studies of overall flushing of confined bodies. The paper describes the uses of continuous, intermittent, and instantaneous injections of tracers. Both radioactive and fluorescent tracers have been used in Swedish coastal studies.

A-10. CEDERWALL, KLAS, Discussion of "Horizontal Surface Discharge of Warm Water Jets," by N. Tamai, R. L. Wiegel, and G. F. Thornberg, Power Division, A.S.C.E., Vol. 97, No. PO1, Jan. 1971, pp. 229-234.

This discussion is essentially the section on thermal discharges from the preceding paper, "Dispersion Phenomena in Coastal Environments."

A-11. SOTIL, C. A., "Computer Program for Slot Buoyant Jets into Stratified Ambient Environments," W. M. Keck Laboratory of Hydraulics and Water Resources, Tech. Memo. No. 71-2, June, 1971, 35 pp.

This technical memorandum closely parallels that by Ditmars, abstracted earlier in this section ("Computer Program for Round Buoyant Jets into Stratified Ambient Environments," Tech. Memo. 69-1).

The principal change is from axisymmetric jets ("round jets") to two-dimensional jets ("slot jets"). Both programs allow the solution of buoyant jet problems for irregular ambient density profiles, rather than being constrained to linear profiles as is the case for the generalized solutions of Fan and Brooks.

This paper gives examples for multi-port outfall diffusers (for waste disposal in the ocean). In deep water a row of ports can be idealized as an equivalent slot jet. In one example, it is shown that the maximum height of plume rise computed by this program is only slightly different from that given by simple plume theory. In another example, the relationship of diffuser length and depth to dilution and submergence is explored. Finally, the sensitivity of the solutions to the value of the entrainment coefficient is studied for one case.

A-12. List, E. J., and Imberger, Jorg, "Turbulent Entrainment in Buoyant Jets and Plumes," J. Hydraulics Div., ASCE, 1973 (in press).

Dimensional reasoning, coupled with published experimental results, is used to show that there is no unique entrainment coefficient for turbulent buoyant jets directed vertically upwards. The behavior of such jets in uniform density environments is found to be governed by an entrainment function and a buoyancy function, both of which are functions of the local jet densimetric Froude number and the local jet spreading rate. For plumes, Batchelor's analysis is used to show that the local Froude number and jet spreading angle are constant. These two facts are then used to show that the buoyancy and entrainment functions in plumes are also constant and that their values can be deduced from an experimental velocity profile alone. The value of the buoyancy function so obtained shows remarkable agreement with the value computed directly from the density profile.

Experimental results are also used to show that the spreading angle of a round vertical buoyant jet is virtually independent of elevation, and this result leads to the conclusion that the entrainment function is linearly dependent on the inverse of the local jet Froude number thus confirming the result obtained by Priestley and Ball (1955). The buoyancy function is found to be constant with the value 1.16 for axially symmetric buoyant jets.

Similar results are derived for two-dimensional buoyant jets, although existing experimental data is inconsistent and therefore inconclusive with respect to predicting the form of the entrainment and buoyancy functions.

These results will allow for improved modelling of round buoyant jet behavior under less restrictive conditions (non-vertical discharge, stratified ambient fluid) by allowing an appropriate transition in the entrainment coefficient (α) from jet-like behavior near the source ($\alpha \sim 0.057$ for round jets) to plume-like behavior far from the source ($\alpha \sim 0.082$ for round plumes).

B. OCEAN OUTFALL DESIGN

B-1. BROOKS, NORMAN H., "Lecture Notes on Conceptual Design of Submarine Outfalls," Univ. of Calif., Berkeley, Water Resources Engineering Educational Series, Program VII, Pollution of Coastal and Estuarine Waters, Jan. 29-30, 1970 (Part I, 25 pp., Part II, 12 pp.) Available as Tech. Memos 70-1 and 70-2, W. M. Keck Laboratory of Hydraulics and Water Resources, Caltech.

Part I deals with the prediction of initial dilution for an ocean outfall by use of various formulas for buoyant jets and plumes. For uniform environment, the cases covered are single round horizontal buoyant jets (open-end pipe); simple round plume (limiting case of low jet velocity for round buoyant jet); and simple line plume (multiport diffuser in deep water). For linearly stratified ambient water, the cases of round and line buoyant plumes are presented, with formulas for predicting height of rise and dilution. An empirical procedure for non-linear environments is derived. For most disposal problems through multiport diffusers in deep water the plume approximations are adequate. Examples are given, with the presumption that the currents are weak.

Part II explains the procedure for designing a multiport diffuser which will distribute the flow well without undue head loss or sea water intrusion. Effects of density difference, bottom slope, and flow variability are included. The procedure is essentially that given by Rawn, Bowerman and Brooks (Trans. ASCE, 126, III, 1961, 344-388), except for the port discharge coefficients which are based on more recent laboratory experiments (not part of this project). The calculation procedure may be easily programmed for computer solution.

B-2. FISCHER*, H. B., AND BROOKS, N. H., "Technical Aspects of Waste Disposal in the Sea through Submarine Outlets", FAO Technical Conference on Marine Pollution and Its Effects on Living Resources and Fishing, Rome, Dec. 1970, Paper FIR: MP/70/R-4 (16 pp.). (*Univ. of California, Berkeley.)

This paper covers the same material on initial dilution as in Brooks (B-1 above). In addition a model is given for predicting further natural dilution and dieoff between outfall and shore. At the end of the paper there are comments on thermal discharges, raising the question of whether it is preferable to have a warm thinner layer than a thick well mixed layer of smaller temperature rise.

C. SELECTIVE WITHDRAWAL AND ARTIFICIAL MIXING IN DENSITY STRATIFIED IMPOUNDMENTS

C-1. BROOKS, NORMAN H., and KOH, ROBERT C. Y., "Selective Withdrawal from Density-Stratified Reservoirs," J. of the Hydraulics Division, ASCE, Vol. 95, No. HY4, Proc. Paper 6702, July, 1969, pp. 1369-1400. (Also in Proc., ASCE Specialty Research Conference on "Current Research into Effects of Reservoirs on Water Quality," Portland, Oregon, Jan. 22-24, 1968, published by Vanderbilt University, Dept. of Environmental and Water Resources Eng., Tech. Report No. 17, pp. 169-214.)

Analyses and experiments for selective withdrawal flows from linearly stratified fluids, including inviscid, viscous, and turbulent cases have been presented. A review of discrete layer systems is also included. These thin jet-like flows have been observed both in the laboratory and in large reservoirs, and boundary layer assumptions appear to be justified.

For turbulent withdrawal flows away from the immediate vicinity of the outlet, it is hypothesized that Koh's viscous diffusive experiments and analysis can be applied by replacing the kinematic viscosity, ν, and the molecular diffusivity, D, by the vertical eddy diffusivity, E_m. For self-generated turbulence, it is predicted that the proper characteristic length is $a = (q/\sqrt{g\varepsilon})^{1/2}$, in which q = the unit discharge and $\varepsilon = -(1/\rho_o)\ d\rho/dy$.

Equations and a graph are given for predicting the thickness of the withdrawal layer under various flow assumptions from real reservoirs. Transients are also presented.

The results of this paper may be used in predicting the performance of a multioutlet system in dams for water quality control by selective withdrawal.

C-2. DITMARS, JOHN D., "Mixing of Density-Stratified Impoundments with Buoyant Jets," W. M. Keck Laboratory of Hydraulics and Water Resources, Report No. KH-R-22, Sept. 1970, 203 pp.

This study was an investigation of the mixing of density-stratified impoundments by means of buoyant jets created by a pumping system. The deterioration of water quality which often occurs in density-stratified lakes and reservoirs may be counteracted by mixing. The physical aspects of the mixing process are the primary concern of this study, although several implications regarding changes in water quality are indicated.

A simulation technique is developed to predict the time-history of changes in the density-depth profiles of an impoundment during mixing. The simulation model considers the impoundment closed to all external influences except those due to the pumping system. The impoundment is treated in a one-dimensional sense, except for the fluid mechanics of the three-dimensional jet and selective withdrawal of pumping system. The numerical solution to the governing equations predicts density profiles at successive time steps during mixing, given the initial density profile, the area-depth relation for the impoundment, the elevations of intake and jet discharge tubes, and the jet discharge and diameter. The changes due to mixing in the profiles of temperature and of a conservative, non-reacting tracer can be predicted also.

The results of laboratory experiments and two field mixing experiments in which density-stratified impoundments were mixed using pumping systems show that the simulation technique predicts the response of the impoundment reasonably well.

The results of a series of simulated mixing experiments for impoundments which have prismatic shapes and initially linear density profiles are given in dimensionless form. For these special conditions, the efficiency of the pumping system increased as the jet densimetric Froude number decreased, and the time required for complete mixing was a fraction of the characteristic time, $T \leq Ꝟ/Q$ (where Ꝟ is the impoundment volume included between intake and jet elevations and Q is the pumped discharge).

Recommendations are made for the application of the generalized results and for the use of the simulation technique for lakes and reservoirs which are not closed systems.

D. NATURAL DIFFUSION IN RESERVOIRS, LAKES, AND OCEANS

D-1. YUDELSON, JERRY M., "A Survey of Ocean Diffusion Studies and Data," W. M. Keck Lab. of Hydraulics and Water Resources, Tech. Memo No. 67-2, Caltech, Sept. 1967, 126 pp.

A study was made of eddy diffusion in the ocean, with emphasis on the dispersion of sewage waste fields.

A thorough search of the literature on ocean diffusion was made, special emphasis being placed on publications of the 1959-1967 period. The object of the search was to review recent theoretical developments and to procure as much observational data as possible, for comparison with several theoretical models of ocean diffusion.

Equations for the dispersion of sewage fields, developed by N. H. Brooks, were compared with many observations of ocean diffusion and found to give good agreement with observed fact. The validity of other equations for ocean diffusion was also discussed.

D-2. SULLIVAN, PAUL J., "Some Data on the Distance-Neighbour Function for Relative Diffusion," Journal of Fluid Mechanics, Vol. 47, Part 3, June 14, 1971, pp. 601-607.

Repeated observations of dye plumes on Lake Huron are interpreted according to the theoretical proposals of Richardson (1926) and Batchelor (1952) about the characteristics of a dispersing cloud of marked fluid within a field of homogeneous turbulence. The results show the average of several instantaneous concentration distributions about their centre of gravity to be approximately Gaussian and the distance-neighbour function to be of approximately Gaussian form. The data are consistent with the theoretical description given by Batchelor, namely,

$$q(y,t) = (2\pi\overline{y^2})^{-1/2}\exp(-y^2/2\overline{y^2}),$$

$$\overline{y^2} = \left(\frac{2}{3}\,\alpha T\right)^3 ,$$

where q(y,t) is the distance-neighbour function and α is the constant of the '4/3-power law'. The average value of α is estimated to be $0 \cdot 12$ cm$^{2/3}$ sec^{-1}. The rate of turbulent energy dissipation in the near-surface currents of Lake Huron is estimated as $\varepsilon \sim 2 \cdot 1 \times 10^{-3}$ cm^2 sec^{-3}.

(The field work was performed as a thesis project by the author at the University of Waterloo; at a later date the preparation of this paper for publication was undertaken as an activity under this grant.)

D-3. RUMER, RALPH R., JR., and HOOPES, JOHN A., "Modelling Great Lakes Circulation," The Water Environment and Human Needs, Symposium, October 1-2, 1970, Parsons Laboratory for Water Resources and Hydrodynamics, M.I.T., pp. 212-247.

Better knowledge of circulation processes in large lakes would enable better predictions of the fate of pollutants discharged into these water bodies. The use of rotating hydraulic models of large lakes in conjunction with the study of mathematical and numerical models and the collection of prototype field data has proven to be a useful method for studying these circulation processes.

This paper reviews the theoretical justification and limitations of rotating laboratory models. Model studies can provide significant information on the large-scale current and seiche motions generated by inflows and outflows to the lakes and by wind stresses imposed at the surface, and on the residence periods for conservative tracers introduced at various points.

(Only the preparation of Rumer's part of this review paper was supported by this research project.)

D-4. RUMER, RALPH R., JR., "Internal Seiches and Interfacial Mixing in Stratified Lakes," W. M. Keck Laboratory of Hydraulics and Water Resources, Tech. Memo 71-3, Caltech, July 1971, 39 pp.

The horizontal velocities of the upper and lower layers associated with an internal seiche episode in a stratified lake are examined in relation to critical shear gradients necessary for the growth of unstable short period interfacial waves with frequency close to the so-called Brunt-Vaisala frequency. Experimental results are presented which help to clarify the conditions for the occurrence of the short period waves. Charts summarizing the findings of this study are presented which should be of help in predicting the occurrence of internal wave breaking in closed basins. It is believed that interfacial wave breaking in a stratified lake increases vertical mixing between the epiliminon and hypoliminon, and therefore may be a factor in the seasonal development of the thermocline and in the distribution of dissolved and suspended substances.

D-5. RUMER, RALPH R., JR., "Interfacial Wave Breaking in Stratified Liquids," J. of Hydraulics Div., ASCE, vol. 99, No. HY3, Mar. 1973, pp. 509-524.

This published paper is a slightly revised version of D-4 above.

E. MIXING IN TURBULENT SHEAR FLOWS

E-1. PRYCH, EDMUND A., "Effects of Density Differences on Lateral Mixing in Open-Channel Flows," W. M. Keck Laboratory of Hydraulics and Water Resources, Report No. KH-R-21, Caltech, May 1970, 225 pp.

This study investigated lateral mixing of tracer fluids in turbulent open-channel flows when the tracer and ambient fluids have different densities. Longitudinal dispersion in flows with longitudinal density gradients was also investigated analytically.

Lateral mixing was studied in a laboratory flume (40 meters long and 110 centimeters wide), by introducing fluid tracers at the ambient flow velocity continuously and uniformly across a fraction of the flume width and over the entire depth of the ambient flow. Fluid samples were taken to obtain concentration distributions in cross-sections at various distances, x, downstream from the tracer source. The data were used to calculate variances of the lateral distributions of the depth-averaged concentration. When there was a difference in density between the tracer and the ambient fluids, lateral mixing close to the source was enhanced by density-induced secondary flows; however, far downstream where the density gradients were small, lateral mixing rates were independent of the initial density difference. A dimensional analysis of the problem and the data show that the normalized variance is a function of only three dimensionless numbers, which represent: (1) the x-coordinate, (2) the source width, and (3) the buoyancy flux from the source.

A simplified set of equations of motion for a fluid with a horizontal density gradient was integrated to give an expression for the density-induced velocity distribution. The dispersion coefficient due to this velocity distribution was also obtained. Using this dispersion coefficient in an analysis for predicting lateral mixing rates in the experiments of this investigation gave only qualitative agreement with the data. However, predicted longitudinal salinity distributions in an idealized laboratory estuary agree well with published data.

The results are applicable to discharges of cooling water (or other buoyant or heavy waste waters) from canals or open-end outfalls into turbulent shear flows such as rivers or homogeneous estuaries. For example, it is possible to predict how rapidly hot-water from a low-velocity canal outlet from a power plant on the bank of a river will spread across the river and be diluted with the river flow.

E-2. OKOYE, JOSEPHAT K., "Characteristics of Transverse Mixing in Open-Channel Flows," W. M. Keck Laboratory of Hydraulics and Water Resources, Report No. KH-R-23, Caltech, November 1970, 269 pp.

The transverse spreading of a plume generated by a point source in a uniform open-channel flow was investigated. A neutrally-buoyant tracer was injected continuously at ambient velocity through a small round source at a point within the flow. Tracer concentration was measured in situ at several points downstream of the source using conductivity probes. Most of the experiments were conducted in the tilting flume which is 40 meters long and 110 centimeters wide.

Tracer concentrations were analyzed in two phases.

In Phase I, the time-averaged concentration was evaluated, its distribution within the plume determined, and characteristic coefficients of transverse mixing calculated. It was shown that the transverse mixing coefficient varied with the distance from the bed and was highest near the water surface where the flow velocity was greatest. In contrast to previous speculation, the ratio of the depth-averaged coefficient of transverse mixing $\overline{D}_z$ to the product of the (bed) shear velocity u_* and the flow depth d was not a constant but depended on the aspect ratio $\lambda = d/W$, where W = flume width. For laboratory experiments $\overline{D}_z/u_*d$ decreased from 0.24 to 0.093 as λ increased from 0.015 to 0.20.

In Phase II, the temporal fluctuation of tracer concentration was studied in three sections. In the first, the intermittency factor technique was used to delineate three regions of the plume cross section: an inner core where the tracer concentration c(t) was always greater than the background C_b; an intermittency region where c(t) was only intermittently greater than C_b; and the outer region where C_b was never exceeded. Dimensional analysis furnished universal curves for prediction of the geometric characteristics of the three regions. In the second section, the entire plume, at a fixed station, was treated as a fluctuating cloud. Variances characterizing the fluctuation of the plume centroid and the variation of the plume width were calculated and compared. In the third section, the intensity and probability density of the concentration fluctuations at fixed points were calculated. The distribution of the peak-to-average ratio was also determined.

Finally the results of the two phases of study were interrelated to evaluate their contributions to the transverse spreading of the plume.

E-3. CEDERWALL, KLAS, "Float Diffusion Study," Water Research, Vol. 5, pp. 889-907, Nov. 1971. (Also W. M. Keck Laboratory of Hydraulics and Water Resources, Tech. Memo. 71-1, April, 1971.)

This paper reviewed the literature on lateral diffusion of solutes and discrete particles in open-channel flow, and presented the results of a new set of experiments with floats. The floats were each constructed of four vanes at right angles to each other and attached to a vertical axis; flotation was arranged so that the vanes were submerged 6 cm below the surface. In contrast to previous float studies, these floats measured the diffusion within the flow, rather than at the free surface. Three sizes were used, measuring 4, 8, and 16 cm respectively for the outside lateral dimensions (tip-to-tip of vanes).

From fifty repeated drops for each of three floats in two flow conditions, the variance of the lateral position of each group was determined as a function of distance downstream; the diffusion coefficient was determined from the rate of change of the variance. It was found that the diffusion coefficient for the particles decreases as the size increases, becoming for the largest floats only 40% of the value for solutes as measured by Okoye in the same flume.

The total rotations of the floats (summed in an absolute sense) were also observed and related to float size. The response of the floats to the vorticity field also decreased with increasing float size, indicating that the larger floats are not affected by the smaller scales of vorticity.

E-4. COUDERT, JEAN F., "A Numerical Solution of the Two-Dimensional Diffusion Equation in a Shear Flow," W. M. Keck Laboratory of Hydraulics and Water Resources Tech. Memo 70-7, Caltech, June, 1970, 41 pp.

A numerical method developed in the last two years was used to solve the two-dimensional diffusion equation with a Dirac δ-function as boundary value, representing a steady line source across a stream. The velocity profile is taken as logarithmic and the diffusivity profile as parabolic, although the method can be used for any measured distribution of velocity and diffusivity. The source can be set anywhere from the bed up to the free surface. The results for each problem are given in a dimensionless form and a significant set of plots is automatically produced. The method can be easily applied to more complicated cases (such as unsteady, point sources).

The numerical technique may be applied to the practical problem of predicting the distance along a river that it takes for a contaminant introduced at the bed to become essentially fully mixed over the depth; the detailed transitional concentration profiles are also developed.

E-5. PRYCH, EDMUND A., Discussion of "Numerical Studies of Unsteady Dispersion in Estuaries," by D. R. F. Harleman, Chok Hung Lee, and L. C. Hall, Jour. of Sanitary Eng. Div., ASCE, Vol. 95, No. SA5, pp. 959-964.

In this discussion it was shown that some numerical estuary models, such as given by the original paper, can accumulate errors which have the same effect as increased dispersion.

Prych's analysis discloses that deficiencies in the finite difference approximation for the convective term in the mass-transport equation may cause much more dispersion in the computations than does the actual dispersion term.

The actual longitudinal dispersion coefficient for the Potomoc Estuary analysis presented by the authors may be ten times what they inferred from their model which inadvertantly included large pseudo-dispersion due to numerical errors. Even verifying an estuary model against field data does not reveal this kind of error, because it is compensated for by reduction in the derived natural dispersion coefficient.

E-6. SULLIVAN, PAUL J., "Longitudinal Dispersion within a Two-Dimensional Turbulent Shear Flow," Journal of Fluid Mechanics, Vol. 49, Pt. 3, 15 Oct. 1971, pp. 551-576.

This paper describes some laboratory and numerical experiments made on the longitudinal dispersion in an open channel flow. Particular attention has been paid to the initial stages of the process.

Physical arguments suggest that the streamwise dispersion of a line of marked fluid elements across a two-dimensional turbulent shear flow occurs in three distinct stages. These stages are identified by a change in the form of the distribution of marked fluid elements in the flow direction. The skewed distribution of the first stage is readily identified by a constant value (approximately 1.1 for the ratio of the peak velocity (V_1) of the distribution to the mean-flow velocity $\overline{U}$; experiments using dyed fluid, made at this stage of the process, have revealed six identifiable features of the suggested distribution. The distributions suggested for the second and the third stage are consistent with the experimental findings of Elder (1959) for the second stage and Taylor (1954) for the third stage.

An attempt has been made to simulate the process numerically using a Markovian model. The results of the simulation confirm features suggested by physical arguments and are in agreement with the open channel experiments.

The results of an experiment, in which the three-dimensional motion of small neutrally buoyant spheres was recorded in many small discrete time intervals, corroborate the theoretical suggestions and simulation results.

(This research was done by the author as a doctoral thesis at Cambridge University prior to his residence at Caltech; the EPA/WQO research grant supported the preparation of this paper for publication.)

F. DISPERSION IN FLOW THROUGH POROUS MEDIA

F-1. LIST, E. JOHN and BROOKS, NORMAN H., "Lateral Dispersion in Saturated Porous Media," J. Geophysical Research, Vol. 72, No. 10, May 15, 1967, pp. 2531-2541.

An analysis of experimental results from a series of lateral dispersion experiments is presented. It is shown that lateral dispersion for low molecular Péclet numbers is adequately described by Saffman's capillary model but that velocity power laws are limited in their application. The dynamic Péclet number is shown to obtain a maximum at molecular Peclet numbers of $O(10^4)$.

(This work was completed under a preceding grant (USPHS/NIH WP-00680) although finally published during this grant period.)

F-2. LIST, E. JOHN, "A Two-Dimensional Sink in a Density-Stratified Porous Medium," J. of Fluid Mechanics, Vol. 33, Part 3, Sept. 2, 1968, pp. 529-543.

A solution is offered for the flow induced by a two-dimensional line sink in a saturated, density-stratified porous medium. It is found that fluid is selectively withdrawn from a thin layer at the elevation of the line sink and not from the entire medium. The velocity distributions predicted by the theory are checked by experiments in a Hele-Shaw cell and good agreement found.

The results of this research may be applied to the problems of selective extraction of one lens of ground water from a layered system, wherein the different layers may have different water qualities and densities.

(This experimental work was completed under a preceding grant (USPHS/NIH WP-00680) and the theoretical development was done by the author while a staff member at Univ. of Auckland; however, the final publication occurred during this grant period.)

F-3. WOODING, ROBIN A., "Growth of Fingers at an Unstable Diffusing Interface in a Porous Medium or Hele-Shaw Cell," J. of Fluid Mechanics, Vol. 39, Part 3, 1969, pp. 477-495. (Also in slightly more detail as W. M. Keck Laboratory of Hydraulics and Water Resources Tech. Memo No. 69-5, Caltech, March 1969, 38 pp.)

Waves at an unstable horizontal interface between two fluids moving vertically through a saturated porous medium are observed to grow rapidly to become fingers (i.e. the amplitude greatly exceeds the wavelength). For a diffusing interface, in experiments using a Hele-Shaw cell, the mean amplitude taken over many fingers grows approximately as $(\text{time})^2$, followed by a transition to a growth proportional to time. Correspondingly, the mean wave-number decreases approximately as $(\text{time})^{-1/2}$. Because of the rapid increase in amplitude, longitudinal dispersion ultimately becomes negligible relative to wave growth.

To represent the observed quantities at large time, the transport equation is suitably weighted and averaged over the horizontal plane. Hyperbolic equations result, and the ascending and descending zones containing the fronts of the fingers are replaced by discontinuities. These averaged equations form an unclosed set, but closure is achieved by assuming a law for the mean wave-number based on similarity. It is found that the mean amplitude is fairly insensitive to changes in wave-number. Numerical solutions of the averaged equations give more detailed information about the growth behaviour, in excellent agreement with the similarity results and with the Hele-Shaw experiments.

The results are applicable to the prediction of convective instabilities in ground water masses when heavier water (colder or more saline) is introduced or recharged above a slightly lighter layer. In large open bodies of water the convective mixing proceeds fairly rapidly when fluids are hydrostatically unstable, because there is little internal resistance. On the other hand, in ground water, there is considerable resistance to convective overturning, and rates are much slower.

G. GENERAL

G-1. BROOKS, NORMAN H., "Thermal Power -- New Crisis for the Environment," Proc. Symposium on Water Environment and Human Needs, Parsons Water Resources and Hydrodynamics Lab., MIT, Oct. 1970, pp. 146-177.

The rapid growth of electric power demand has led to a crisis in power-plant siting because of the substantial environmental effects of large thermal power stations. The water environment has been the most convenient receptor for the huge discharges of waste heat. Inland lakes and rivers have already been used or committed to their tolerable limits of temperature, and marine and estuarine discharges are raising difficult questions of long-range strategies and effects.

This survey paper will cover the following topics: magnitude of the waste heat problem, now and projected for the future; hydraulic and hydrologic aspects of thermal discharges; alternative strategies and costs for power-plant siting (e.g. inland plants with cooling towers vs. coastal plants); some broad issues for society relating to choices between environmental alternatives and stopping the rapid growth of energy usage; and implications for urban planning.

Some research results by Prych (E-1 above) on accelerated transverse mixing of a buoyant fluid in an open channel flow are included.

G-2. BROOKS, NORMAN H., Discussion of paper "The Mechanics of Thermally Stratified Flow" by D. R. F. Harleman, Proc. National Symposium on Thermal Pollution (1968), Vanderbilt Univ. Press, 1969, pp. 165-172.

The anology of thermal outfalls to sewer outfalls is discussed, especially with reference to the possibility of using multiple-jet diffusers to increase the initial dilution for discharges from thermal outfalls (lower ΔT). Submergence of waste heat below thermoclines is also possible in large bodies of water.

This strong mixing approach can be expected to be used in the ocean because of the large heat capacity and less danger of interfering with annual cycles of thermal stratification and overturning.

The reader is cautioned against using the common jet and plume formulas, which presume infinite flow fields, in cases of restricted depths, i.e. situations where restricted access of diluting water may limit dilutions obtained. Hydraulic model studies, as described by Harleman, are needed in such cases.